PRESENT AND FUTURE ASPECTS OF CP VIOLATION

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ABSTRACT

This series of five lectures describes aspects of CP violation, emphasizing its description within the standard electroweak model. After discussing the kaon system, the only place in which CP violation has been seen so far, we turn to the leading contender for the effect, complex phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. A number of suggestions are made for improved tests of the standard picture. Hadrons containing b quarks play a key role in this program, and are discussed separately. We also mention a number of non-standard and speculative aspects of CP violation, including alternatives to the CKM description, direct tests of time reversal invariance, and baryogenesis.

1. INTRODUCTION

A.   Overview of CP violation

For many years, it was widely believed that the laws of physics were invariant under the separate discrete symmetry operations of spatial reflection (*parity,* or P), time reversal (T), and charge conjugation (C). However, it was only possible to prove, under the assumptions of locality and Lorentz-invariance, that a quantum field theory must respect the product CPT.[1] In 1956 it was realized that no tests of P or C invariance of the weak interactions had yet been performed.[2] Experiments soon showed that both P and C were separately violated in weak decays.[3]

The theory of weak interactions as formulated[4] in 1957 did conserve the product CP. Seven years later, the discovery of the two-pion decay of the neutral kaon[5] showed that even the product CP was violated. Since 1964, although no new CP-violating phenomena have been observed, we have a theory for this effect and the prospect of many experimental tests. These lectures describe our present understanding of CP violation, some of the tests which are likely to bear fruit in the near future, and other ways in which CP violation can manifest itself.

B.   Plan of these lectures

Section 2 is devoted to a description of CP violation in the kaon system, without regard to its fundamental origin. A candidate theory for CP violation based on complex phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is described in Section 3. Prospects for improving information about that theory (or disproving it altogether by exposing inconsistencies) are mentioned in Section 4. A crucial role is played by B mesons (particularly by CP violation in their decays), to which Section 5 is addressed. Non-standard and speculative aspects of CP violation are the subject of Section 6, while Section 7 concludes.

C.   A short bibliography

A good overall reference on quantum field theory, with an excellent section on CP violation, is the textbook by Cheng and Li.[6] There are many fine articles on CP violation in a recent World Scientific publication edited by Cecilia Jarlskog.[7]

For historical perspectives I recommend the proposals for a short- and long-lived neutral kaon;[8,9] the original report of the discovery of CP violation;[5] early and later reviews of CP violation in the neutral kaon system;[10] and original articles on the Cabibbo theory,[11] charmed quarks,[12] and the third

family of quarks.[13] An article which was of tremendous use in anticipating properties of the charmed quark is the study of kaon decays by Gaillard and Lee.[14]

We shall make frequent use of a parametrization of quark couplings,[15] updating earlier analyses[16] in which data constrain these parameters. More general possibilities for the study of CP violation have arisen since the suggestion[17] that these effects may be large for mesons containing b quarks. The reader is invited to consult reviews of earlier developments in this area.[18]

For aspects of CP violation not confined to the kaon and B meson systems, one may consult a review[19] of the strong CP problem[20] and one proposed solution of it,[21] as well as the paper of Sakharov[22] which proposed that CP violation was one of the ingredients responsible for the observed baryon asymmetry of the Universe.

The present lectures are based in part on earlier treatments,[23–28] updated to take account of recent developments. In some cases these references may be consulted for greater detail.

2. CP VIOLATION IN THE KAON SYSTEM

A. CP Eigenstates of neutral kaons

In order to make sense of a class of "strange" particles produced strongly but decaying weakly, Gell-Mann and Nishijima[29] proposed an additive quantum number, "strangeness," conserved in the strong interactions but not necessarily in the weak interactions. The reaction $\pi^- p \to K^0 \Lambda$, for example, would conserve strangeness S if $S(\pi) = S(p) = 0$, $S(\Lambda) = -1$, and $S(K^0) = +1$. But then the kaon could not be its own antiparticle; there would have to also exist a $\bar{K}^0$ with $S(\bar{K}^0) = -1$. It could be produced, for example, in the reaction $\pi^- p \to K^0 \bar{K}^0 n$.

The states K^0 and $\bar{K}^0$ would be degenerate in the absence of coupling to final states (or to one another). However, both states can decay to the 2π final state in an S-wave (orbital angular momentum $\ell = 0$). Gell-Mann and Pais[8] noted that since $C(\pi^+\pi^-)_{\ell=0} = +$, $C(K^0) = \bar{K}^0$, and $C(\bar{K}^0) = K^0$, the linear combination of K^0 and $\bar{K}^0$ which decayed to $\pi^+\pi^-$ had to be $K_1 \equiv (K^0 + \bar{K}^0)/\sqrt{2}$. Then there should be another state $K_2 \equiv (K^0 - \bar{K}^0)/\sqrt{2}$ forbidden to decay to $\pi^+\pi^-$, and thus long-lived. (It should be able to decay to 3π, for example.) This state was looked for and found.[30] Its lifetime was measured to be about 600 times that of K_1.

The $K^0 - \bar{K}^0$ system resembles many coupled degenerate problems in physics. For example, a drum-head in its first excited state possesses a line of nodes. A degenerate state exists with the line of nodes perpendicular to the first, but nothing specifies the absolute orientations of the two lines of nodes. However, if a fly alights off-center on the drum, it will define the two lines of nodes. One mode will couple to the fly, thereby changing in frequency, and one mode will not.

Gell-Mann and Pais assumed that C was conserved in the weak decay process. In a CP-conserving weak interaction theory in which C and P are individually violated, the above argument can be recovered by replacing C with CP.[9]

The discovery[5] in 1964 that both the short-lived "K_1" and long-lived "K_2" states decayed to $\pi\pi$ upset this tidy picture and signified that not even CP symmetry was valid in Nature. Henceforth the states of definite mass and lifetime would be known as K_S (for "short") and K_L (for "long"). Before describing the manifestations of CP violation in neutral kaon decays, we discuss briefly the CP properties of final states containing pions.

B. CP properties of 2π and 3π states

1. *Two-pion states.* A $\pi^+\pi^-$ state of relative orbital angular momentum ℓ has charge conjugation eigenvalue $C = (-1)^\ell$ and parity $P = (-1)^\ell$, so it is always an eigenstate of CP with eigenvalue $+1$. For a $\pi^0\pi^0$ state one must have $C = P = +$, and only even ℓ are allowed.

2. *Three-pion states.* We shall concentrate on neutral states with total angular momentum $J = 0$. Consider first the $\pi^+\pi^-\pi^0$ state, with the relative orbital singular momentum of π^+ and π^- defined to be ℓ_{12}, and that of the π^0 with respect to the $\pi^+\pi^-$ system taken to be ℓ_3. Since

$J = 0$, one must have $\ell_3 = \ell_{12}$. The intrinsic CP of the $\pi^+\pi^-$ system is $+$, and that of π^0 is $-$, since $C(\pi^0) = +$, $P(\pi^0) = -$. Then

$$CP(\pi^+\pi^-\pi^0)|_{J=0} = -(-1)^{\ell_3} = (\text{odd, even, odd}, \ldots) \text{ for } \ell_3 = (0, 1, 2, \ldots) \tag{1}$$

The corresponding isospins allowed for the $\pi^+\pi^-$ system are (even, odd, even, ...). (Recall $\ell_{12} = \ell_3$). The states with $\ell_{12} = \ell_3 > 0$ are highly suppressed in kaon decays by centrifugal barrier effects. Thus in $J = 0$ states of $\pi^+\pi^-\pi^0$, the CP-odd state predominates. The situation is simpler for the $3\pi^0$ states. Here ℓ_{12} (and hence ℓ_3) must be even, so *only* the CP-odd states can occur. The 3π states are the dominant hadronic decay mode of the K_L.

C. The $\Delta I = 1/2$ rule

The nonleptonic decays of strange particles are governed by the quark subprocess $s \to ud\bar{u}$, which produces three $I = 1/2$ quarks in the final state and thus can lead to $\Delta I = 1/2$ and $\Delta I = 3/2$. In almost all processes, $\Delta I = 1/2$ dominates. The reason for this involves a combination of several effects, as we shall see for $K \to 2\pi$ decays.

The isospin states $|I\rangle$ of two pions with momenta p_1 and p_2 may be expressed in terms of the charge states $|Q(\pi[p_1])Q(\pi[p_2])\rangle$, where $Q = +, 0$, or $-$, as

$$|0\rangle = [|+-\rangle - |0\ 0\rangle + |-+\rangle]/\sqrt{3} \quad (J = \text{ even}) \quad, \tag{2}$$

$$|1\rangle = [|+-\rangle - |-+\rangle]/\sqrt{2} \quad (J = \text{ odd}) \quad, \tag{3}$$

$$|2\rangle = [|+-\rangle + 2|0\ 0\rangle + |-+\rangle]/\sqrt{6} \quad (J = \text{ even}) \quad. \tag{4}$$

Inverting these relations, we find, for even J (the case of interest in neutral kaon decay)

$$\langle\pm\mp|T|K\rangle = \langle 0|T|K\rangle/\sqrt{3} + \langle 2|T|K\rangle/\sqrt{6} \quad, \tag{5}$$

$$\langle 0\ 0|T|K\rangle = -\langle 0|T|K\rangle/\sqrt{3} + \langle 2|T|K\rangle\sqrt{2/3} \quad. \tag{6}$$

To take into account identical particles we integrate over phase space with $\hat{p}_1$ in one hemisphere and $\hat{p}_2$ in the other, taking both $\langle\pi^+(p_1)\pi^-(p_2)|$ and $\langle\pi^-(p_1)\pi^+(p_2)|$ into account. For the $\pi^+\pi^-$ final state, this avoids double counting in $\langle\pi^0(p_1)\pi^0(p_2)|$. We then find that if $I_{\pi\pi} = 0$ is dominant, $\Gamma(K^0 \to \pi^+\pi^-) = 2\Gamma(K^0 \to \pi^0\pi^0)$, while if $I_{\pi\pi} = 2$, $\Gamma(K^0 \to \pi^0\pi^0) = 2\Gamma(K^0 \to \pi^+\pi^-)$. Experimentally the ratio[31] is much closer to that for $I_{\pi\pi} = 0$. The small deviation from a $2:1$ ratio for $\pi^\pm\pi^\mp : \pi^0\pi^0$ is due to Coulomb corrections and to a small $I_{\pi\pi} = 2$ admixture in the amplitude, whose magnitude may be estimated by comparing the rate for $K^+ \to \pi^+\pi^0$ (which must involve $I_{\pi\pi} = 2$; $I_{\pi\pi} = 1$ is forbidden by Bose statistics) with that for $K_S^0 \to \pi\pi$:

$$\Gamma(K^+ \to \pi^+\pi^0) = B(K^+ \to \pi^+\pi^0)/\tau_{K^+} = 1.71 \times 10^7 s^{-1} \tag{7}$$

$$\Gamma(K_S^0 \to \pi\pi) \simeq 1/\tau_{K_S} = 1.12 \times 10^{10} s^{-1} \tag{8}$$

The $K_S \to \pi\pi$ decay rate is more than 600 times as large as the $K^+ \to \pi^+\pi^0$ rate.

Several mechanisms probably contribute to the enhancement of the $\Delta I = 1/2$ transition (leading to $I_{\pi\pi} = 0$) with respect to the $\Delta I = 3/2$ transition (leading to $I_{\pi\pi} = 2$). They include the following:

1. The "penguin graph"[32] involves the transition $s \to d$ with emission of at least one gluon. (The transition without gluon emission corresponds to a term which can be removed from the Lagrangian by a small redefinition of quark fields.[33]) Since a single d quark is created, the penguin graph automatically has $\Delta I = 1/2$.

2. The "exchange graph", corresponding to the process $s\bar{d} \to u\bar{u}$ mediated by W exchange, would lead to a $u\bar{u}$ final state if applied to an initial neutral kaon. A $u\bar{u}$ state cannot have $I = 2$, so this graph could also play a role in enhancement of $K \to (2\pi)_{I=0}$. However, it is thought to be

an unlikely contributor to decays involving light quarks in a state of $J = 0$. In the limit of vanishing quark mass, the s quark should participate in the process with left-handed helicity, while the $\bar{d}$ should participate with right-handed helicity. The total z component of spin of the s and $\bar{d}$ quarks would then be -1 along the direction of the s quark in the $s\bar{d}$ c.m.s., which is impossible for a $J = 0$ state like the kaon. Perturbative QCD effects appear inadequate to overcome the expected suppression.[34]

    3. Final-state interactions may well contribute to the enhancement of $I_{\pi\pi} = 0$. The $\pi\pi$ phase shift δ_I is large and positive ($\approx 45^0$) for $I_{\pi\pi} = 0$, but small and negative ($\approx -7^0$) for $I_{\pi\pi} = 2$. Resonant behavior would correspond to $\delta_I = \pi/2$.

    4. Perturbative QCD effects undoubtedly enhance $\Delta I = 1/2$ transitions. The effective weak Hamiltonian for $s \to ud\bar{u}$ can be decomposed into pieces symmetric ($I_{ud} = 1$) and antisymmetric ($I_{ud} = 0$) under the interchange of the final u and d quarks. The $I_{ud} = 0$ piece receives an enhancement from perturbative QCD, while the $I_{ud} = 1$ piece is slightly suppressed.[35] The $I_{ud} = 0$ piece contributes only to $\Delta I = 1/2$ amplitudes, since the only other quark carrying isospin in the process is a $\bar{u}$ quark.

    The actual enhancement of $\Delta I = 1/2$ transitions in K decays appears to be due to a combination of several of the effects mentioned above.[36]

D.   Mass eigenstates of short- and long-lived kaons

    In order to discuss neutral K decays (and, later, neutral B decays), we need information on the matrix $\mathcal{M}$ whose eigenstates correspond to particles of definite mass and lifetime.[6,10] In the kaon rest frame, the time evolution of basis states K^0 and $\bar{K}^0$ can be written[37] as

$$i\frac{\partial}{\partial t}\begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = \mathcal{M}\begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} \quad ; \quad \mathcal{M} = M - i\Gamma/2 \quad . \tag{9}$$

An arbitrary matrix $\mathcal{M}$ can be written in terms of Hermitian matrices M and Γ. CPT invariance can be shown to imply the restriction $\mathcal{M}_{11} = \mathcal{M}_{22}$ and hence $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$. We shall adopt this limitation in all our subsequent discussions; one may consult several reviews[10] for the case of CPT violation.

    We denote the eigenstates of $\mathcal{M}$ by

$$|S\rangle + p_S|K^0\rangle + q_S|\bar{K}^0\rangle, \tag{10}$$

$$|L\rangle + p_L|K^0\rangle + q_L|\bar{K}^0\rangle, \tag{11}$$

with $|p_{S,L}|^2 + |q_{S,L}|^2 = 1$, and the corresponding eigenvalues by $\mu_{S,L} \equiv m_{S,L} - \frac{i}{2}\Gamma_{S,L}$, where $m_{S,L}$ and $\Gamma_{S,L}$ are real. Then

$$\mathcal{M}\begin{bmatrix} p_S & p_L \\ q_S & q_L \end{bmatrix} = \begin{bmatrix} p_S & p_L \\ q_S & q_L \end{bmatrix}\begin{bmatrix} \mu_S & 0 \\ 0 & \mu_L \end{bmatrix} \quad , \tag{12}$$

implying

$$\left(\frac{p_S}{q_S}\right)^2 = \frac{\mathcal{M}_{12}}{\mathcal{M}_{21}} = \left(\frac{p_L}{q_L}\right)^2 \tag{13}$$

when the condition $\mathcal{M}_{11} = \mathcal{M}_{22}$ is taken into account. Thus there are two solutions for p's and q's. We choose $p_S = p_L \equiv p$; then $q \equiv q_S = -q_L$, and the eigenstates are

$$|S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle, \tag{14}$$

$$|L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle, \tag{15}$$

or with $\epsilon \equiv (p - q)/(p + q)$,

$$|S\rangle = \frac{1}{\sqrt{2(1 + |\epsilon|^2)}}\left[(1 + \epsilon)|K^0\rangle + (1 - \epsilon)|\bar{K}^0\rangle\right] \quad , \tag{16}$$

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$$|L\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle \right] \quad . \tag{17}$$

In a CPT-invariant theory, a single complex parameter ϵ specifies the eigenstates.

One can relate ϵ more directly to the properties of the mass matrix and mass eigenvalues. Making a phase choice when taking the square root of (13), we write

$$\frac{q}{p} = \sqrt{\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}} \tag{18}$$

and note from (12) that

$$\mu_S = \mathcal{M}_{11} + \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \quad ; \quad \mu_L = \mathcal{M}_{11} - \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \quad , \tag{19}$$

so

$$\mu_S - \mu_L = 2\sqrt{\mathcal{M}_{12}\mathcal{M}_{21}} \quad . \tag{20}$$

Then

$$\epsilon = \frac{p-q}{p+q} = \frac{\sqrt{\mathcal{M}_{12}} - \sqrt{\mathcal{M}_{21}}}{\sqrt{\mathcal{M}_{12}} + \sqrt{\mathcal{M}_{21}}} \simeq \frac{\mathcal{M}_{12} - \mathcal{M}_{21}}{4\sqrt{\mathcal{M}_{12}\mathcal{M}_{21}}} \quad , \tag{21}$$

where the smallness of ϵ has been used. With the definition of $\mathcal{M}$ and (20) we can then write

$$\epsilon \simeq \frac{\mathrm{Im}(\Gamma_{12}/2) + i \, \mathrm{Im}M_{12}}{\mu_S - \mu_L}, \tag{22}$$

so that the CP-violation parameter ϵ arises from imaginary parts of off-diagonal terms in the mass matrix.

The matrices Γ and M may be expressed[38] in terms of sums over states connected to K^0 and $\bar{K}^0$ by the weak Hamiltonian H_W:

$$\Gamma_{12} = 2\pi \sum_F \rho_F \langle K^0|H_W|F\rangle\langle F|H_W|\bar{K}^0\rangle, \tag{23}$$

where ρ_F denotes the density of final states F, and

$$M_{12} = \langle K^0|H_W|\bar{K}^0\rangle + \sum_n \frac{\langle K^0|H_W|n\rangle\langle n|H_W|\bar{K}^0\rangle}{m_{K^0} - m_n} \tag{24}$$

By considering specific 2π, 3π, $\pi l\nu$, and other final states, one can show that $|\mathrm{Im}\Gamma_{12}/2| \ll |\mathrm{Im}M_{12}|$, and so

$$\epsilon \simeq \frac{i \, \mathrm{Im}M_{12}}{\mu_S - \mu_L} \quad . \tag{25}$$

This result implies a specific phase of ϵ:

$$\mathrm{Arg} \, \epsilon \approx \left\{ \begin{array}{c} 90^0 \\ 270^0 \end{array} \right\} - \mathrm{Arg}(\mu_S - \mu_L) \quad \text{for} \quad \left\{ \begin{array}{c} \mathrm{Im}M_{12} > 0 \\ \mathrm{Im}M_{12} < 0 \end{array} \right\} \tag{26}$$

Given the measurements:[31]

$$m_S - m_L = -0.476 \, \Gamma_S; \quad \Gamma_S - \Gamma_L = 0.998 \, \Gamma_S, \tag{27}$$

we have $\mu_S - \mu_L = -(0.476 + 0.499i)\Gamma_S$, or $\mathrm{Arg} \, (\mu_S - \mu_L) = (3\pi/2) - \arctan(0.476/0.499) = (3\pi/2) - 43.6^0$. Thus

$$\mathrm{Arg} \, \epsilon = (43.6 \pm 0.2)^0 \quad (\mathrm{Im} \, M_{12} < 0) \quad ,$$

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$$\text{Arg } \epsilon = \pi + (43.6 \pm 0.2)^0 \quad (\text{Im } M_{12} > 0) \quad . \tag{28}$$

E. CP-violating observables and expected phases

The parameter ϵ actually depends on the phase convention used to relate K^0 to $\bar{K}^0$. Observable convention-independent quantities can be defined in terms of ratios of decay amplitudes. We define

$$\epsilon_0 \equiv \frac{\langle 0|T|L\rangle}{\langle 0|T|S\rangle} \quad ; \quad \epsilon_2 \equiv \frac{1}{\sqrt{2}} \frac{\langle 2|T|L\rangle}{\langle 0|T|S\rangle}, \tag{29}$$

which relate CP-violating decays of K_L to 2π in $I = 0$ and $I = 2$ final states to the CP-conserving decay $K_S \to (2\pi)_{I=0}$, and the ratio of CP-conserving $I = 2$ and $I = 0$ amplitudes

$$\omega \equiv \frac{\langle 2|T|S\rangle}{\langle 0|T|S\rangle} \quad . \tag{30}$$

The ratios for specific charge states corresponding to CP-forbidden and CP-allowed 2π decays are defined as

$$\eta_{+-} \equiv \frac{\langle +-|T|L\rangle}{\langle +-|T|S\rangle} \quad ; \quad \eta_{00} \equiv \frac{\langle 00|T|L\rangle}{\langle 00|T|S\rangle} \quad . \tag{31}$$

Recalling the expressions for $|+-\rangle$ and $|00\rangle$ in terms of isospin states, and substituting, we find

$$\eta_{+-} = \frac{\langle 2|T|L\rangle + \sqrt{2}\langle 0|T|L\rangle}{\langle 2|T|S\rangle + \sqrt{2}\langle 0|T|S\rangle} \quad ,$$

$$\eta_{00} = \frac{\sqrt{2}\langle 2|T|L\rangle - \langle 0|T|L\rangle}{\sqrt{2}\langle 2|T|S\rangle - \langle 0|T|S\rangle} \quad , \tag{32}$$

or

$$\eta_{+-} = \frac{\epsilon_0 + \epsilon_2}{1 + \omega/\sqrt{2}} \approx \epsilon_0 + \epsilon' \tag{33}$$

$$\eta_{00} = \frac{\epsilon_0 - 2\epsilon_2}{1 - \omega\sqrt{2}} \approx \epsilon_0 - 2\epsilon' \tag{34}$$

where

$$\epsilon' \equiv \epsilon_2 - \frac{\omega\epsilon_0}{\sqrt{2}} \quad . \tag{35}$$

In order to relate these results to an expression involving ϵ, we must first discuss phases of amplitudes. We may factor out the final state phase shift δ_I in the amplitude $\langle I|T|K^0\rangle$ to write

$$\langle I|T|K^0\rangle \equiv A_I e^{i\delta_I} \quad . \tag{36}$$

Applying CPT to this result, we shall now show that

$$\langle I|T|\bar{K}^0\rangle = A_I^* e^{i\delta_I} \quad , \tag{37}$$

so that the same final state phase appears, accompanied by the complex conjugate amplitude. The proof goes as follows.[39] Let a final state F_{out} be related to a "standing-wave" state by $F_{\text{out}} = e^{i\delta} F_0$. Then, since CPT is an antiunitary operator, CPT $F_{\text{out}} = e^{-i\delta} F_0$. But also CPT $F_{\text{out}} = F_{\text{in}}$, so we have shown that

$$F_{\text{in}} = e^{-2i\delta} F_{\text{out}} \quad . \tag{38}$$

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This relation holds for eigenstates of the S-matrix (such as 2π states with $\mathcal{M}_{\pi\pi} = \mathcal{M}_k$ and $J = 0$). Let us start with $\langle F_{\text{out}} | T | K^0 \rangle = A_I e^{i\delta_I}$. Now consider the desired matrix element:

$$\langle F_{\text{out}} | T | \bar{K} \rangle = \langle CPT\ F_{\text{out}} | T | CPT\ \bar{K} \rangle^* \tag{39}$$

(CPT is antiunitary)

$$= \langle F_{\text{in}} | T | K \rangle^* \tag{40}$$

(by definition of F_{in} and K)

$$= \langle F_{\text{out}} | T | K \rangle^* e^{2i\delta_I} = (A_I e^{i\delta_I})^* e^{2i\delta_I} = A_I^* e^{i\delta_I} \quad, \tag{41}$$

which was to be proved.

We may now substitute the expressions for $|S\rangle$ and $|L\rangle$ into the definitions (29) and (30). First of all, we have

$$\epsilon_0 = \frac{\langle 0 | T | K^0 \rangle (1+\epsilon) - \langle 0 | T | \bar{K}^0 \rangle (1-\epsilon)}{\langle 0 | T | K^0 \rangle (1+\epsilon) + \langle 0 | T | \bar{K}^0 \rangle (1-\epsilon)} = \frac{i \operatorname{Im} A_0 + \epsilon \operatorname{Re} A_0}{\operatorname{Re} A_0 + i \epsilon \operatorname{Im} A_0} \quad . \tag{42}$$

The relation between ϵ and ϵ_0 depends on the phase of A_0. For now, we shall adopt a definition of K^0 and $\bar{K}^0$ such that $\operatorname{Im} A_0 = 0$ (the Wu-Yang[40] convention), in which case $\epsilon_0 = \epsilon$. If, instead, we were to take $A_0 = \rho_0 e^{i\phi_0}$ (ρ_0 real), then

$$\epsilon_0 = \frac{i \sin \phi_0 + \epsilon \cos \phi_0}{\cos \phi_0 + i \epsilon \sin \phi_0} \approx \epsilon(1 - i\phi_0) + i\phi_0 \tag{43}$$

to lowest order in small quantities. This result implies that

$$\operatorname{Re} \epsilon_0 \simeq \operatorname{Re} \epsilon + \phi_0 \operatorname{Im} \epsilon \tag{44}$$

Here the second term is small in comparison with the first if ϕ_0 is small, which it usually is in most conventions differing from the Wu-Yang convention.[41]

The corresponding results for ϵ_2 and ω involve the difference of final state phases δ_2 and δ_0 in $I_{\pi\pi} = 2$ and $I_{\pi\pi} = 0$ channels:

$$\epsilon_2 = \frac{1}{\sqrt{2}} \frac{i \operatorname{Im} A_2 + \epsilon \operatorname{Re} A_2}{\operatorname{Re} A_0 + i \epsilon \operatorname{Im} A_0} e^{i(\delta_2 - \delta_0)} \quad, \tag{45}$$

$$\omega = \frac{\operatorname{Re} A_2 + i \epsilon \operatorname{Im} A_2}{\operatorname{Re} A_0 + i \epsilon \operatorname{Im} A_0} e^{i(\delta_2 - \delta_0)} \quad . \tag{46}$$

When $\operatorname{Im} A_0 = 0$, taking account of (35), we have the simple result

$$\epsilon' = \frac{i}{\sqrt{2}} \frac{\operatorname{Im} A_2}{A_0} e^{i(\delta_2 - \delta_0)} \tag{47}$$

The phase of ϵ' is then governed entirely by the final state $\pi\pi$ phases. We shall show that, just as for ϵ, the phase of ϵ' should be close to $45°$.

Present information on $\pi\pi$ phase shifts[42] for $I_{\pi\pi} = 0$ is based at very low $\mathcal{M}_{\pi\pi}$ on K_{e4} decay,[43] and at higher $\mathcal{M}_{\pi\pi}$ on data from the reaction $\pi^- p \to \pi^+ \pi^- n$ using pion exchange.[44] It is estimated[42] that $\delta_0 - \delta_2 = (42 \pm 4)°$, and

$$\operatorname{Arg} \epsilon' = 48 \pm 4° \quad, \tag{48}$$

which implies [cf. Eq. (28)] that ϵ and ϵ' have nearly the same phase.

Combining $\epsilon_0 = \epsilon$ with Eqs. (32), we have

$$\eta_{+-} = \epsilon + \epsilon' \quad; \quad \eta_{00} = \epsilon - 2\epsilon' \quad . \tag{49}$$

In view of the near-equality of the phases of ϵ and ϵ', we find

$$|\eta_{+-}| \simeq |\epsilon|[1 + \text{Re}(\epsilon'/\epsilon)] \quad ,$$

$$|\eta_{00}| \simeq |\epsilon|[1 - 2\text{Re}(\epsilon'/\epsilon] \quad , \tag{50}$$

so

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\Gamma(K_L \to 2\pi^0)}{\Gamma(K_S \to 2\pi^0)}\Bigg/\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = 1 - 6 \text{ Re}\frac{\epsilon'}{\epsilon} \quad . \tag{51}$$

This double ratio is measurable with considerably less systematic error than any individual ratios involving a single type of decaying particle or single type of final state. Two experiments, one at Fermilab (E731)[45] and one at CERN (NA31),[46] have provided the most recent values, implying

$$\text{NA31}: \quad \text{Re}(\epsilon'/\epsilon) = (23.0 \pm 6.5) \times 10^{-4} \quad , \tag{52}$$

$$\text{E731}: \quad \text{Re}(\epsilon'/\epsilon) = (7.4 \pm 6.0) \times 10^{-4} \quad . \tag{53}$$

The central values obtained by the two experiments are very different in their implications for fundamental theories of CP violation. If $\epsilon' \neq 0$, as suggested by the NA31 result, CP violation must be occurring in a decay amplitude (specifically, via $\text{Im}A_2 \neq 0$) as well as through the mass matrix. If $\epsilon' = 0$ and only ϵ is nonzero, as suggested by the E731 result, CP violation could arise from any of a number of sources, including a special $\Delta S = 2$ "superweak" interaction concocted especially to affect the mass matrix,[47] many other types of interactions to be discussed in Section 6, or even an accidental cancellation of effects within the context of CKM physics. We shall discuss this possibility in Section 4. More data will be forthcoming from both groups.

The results just mentioned suggest that $0 \le \text{Re}(\epsilon'/\epsilon) \le 2 \times 10^{-3}$. With $\text{Arg}(\epsilon'/\epsilon) = (4.3 \pm 4)^0$, one then finds that $\phi_{00} \equiv \text{Arg}(\eta_{00})$ and $\phi_{+-} \equiv \text{Arg}(\eta_{+-})$ should differ by no more than 0.1^0. Anything more implies a breakdown in the logic leading to Eqs. (28) and (48), which would be most naturally blamed on a violation of CPT invariance.

Recently, Fermilab experiment E773[48] has obtained the result $\phi_{00} - \phi_{+-} = (0.62 \pm 1.03)^\circ$, compatible with CPT invariance. A combined fit to E731 and E773 data also finds $\phi_{+-} = (43.53 \pm 0.97)^\circ$, consistent with the value (28) expected for the phase of ϵ and with the world average[31] $\phi_{+-} = (44.3 \pm 0.8)^\circ$.

F.   Semileptonic decays

The charge asymmetry in the semileptonic decays of K_L provides further evidence that it is not a CP eigenstate. Let us define

$$\delta_l \equiv \frac{\Gamma(K_L \to \pi^- l^+ \nu) - \Gamma(K_L \to \pi^+ l^- \bar{\nu})}{\Gamma(K_L \to \pi^- l^+ \nu) + \Gamma(K_L \to \pi^+ l^- \bar{\nu})} \quad . \tag{54}$$

In the standard model of weak charge-changing interactions, only the K^0 component of the K_L leads to $\pi^- l^+ \nu$, since the corresponding quark subprocess is $\bar{s} \to \bar{u} l^+ \nu$. Similarly, only the $\bar{K}^0$ component leads to $\pi^+ l^- \bar{\nu}$; at the quark level this decay is $s \to u l^- \bar{\nu}$. These processes have ΔQ (the change in charge of the hadron) equal to ΔS (the change in its strangeness). Processes with $\Delta Q = -\Delta S$, such as $\bar{K}^0 \to \pi^- l^+ \nu$, have not been observed and are not expected in the standard model. Neglecting their contribution (one may also include them in a more complete treatment[49]), we may use the expression for K_L in terms of K^0 and $\bar{K}^0$ to find

$$\delta_l \simeq 2 \text{ Re } \epsilon \quad . \tag{55}$$

Experimental averages[31] for this quantity are

$$\delta_e = (3.33 \pm 0.14) \times 10^{-3} \quad ; \quad \delta_\mu = (3.04 \pm 0.25) \times 10^{-3} \quad ,$$

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leading to an overall average

$$\delta_l = (3.27 \pm 0.12) \times 10^{-3} \quad . \tag{56}$$

Using $|\epsilon| \simeq |\eta_{+-}|[1 - \text{Re}\,(\epsilon'/\epsilon)] = (2.26 \pm 0.02) \times 10^{-3}$ and Arg $\epsilon = (44.8 \pm 0.8)°$ from the experimental world average,[31] one obtains $2\,\text{Re}\,\epsilon = (3.24 \pm 0.07) \times 10^{-3}$, in excellent accord with Eq. (56).

G. K_S decays

1. The decay $K_S \to 3\pi^0$ is purely CP violating. The CP of the initial state is positive, that of each pion is negative, and the zero spin of K_S and each pion leads to positive parity of the final 3π spatial wave function.

By searching for interference between $K_L \to 3\pi^0$ and $K_S \to 3\pi^0$, it has been found possible[50] to place the indirect bound $B(K_S \to 3\pi^0) < 3.7 \times 10^{-5}$. If this decay were to occur via mixing alone, one would expect

$$B(K_S \to 3\pi^0) = \frac{\tau_S}{\tau_L}|\epsilon|^2 B(K_L \to 3\pi^0) \simeq 2 \times 10^{-9} \quad . \tag{57}$$

2. The decay $K_S \to \pi^+\pi^-\pi^0$) has been studied in the CPLEAR[51] and Fermilab E621[52] experiments. CPLEAR can directly "tag" the flavor of the produced neutral kaon in the reaction $\bar{p}p \to [K^0 K^-\pi^+ \text{ or } \bar{K}^0 K^+\pi^-]$, thereby measuring both η_{+-} (though not, so far, with smaller errors than other experiments) and $\eta_{+-0} \equiv A(K_S \to [\pi^+\pi^-\pi^0]|\text{CP}=-)/A(K_L \to \pi^+\pi^-\pi^0)$. By searching for $K_S - K_L$ interference in the $\pi^+\pi^-\pi^0$ final state, the bound $B(K_S \to \pi^+\pi^-\pi^0) < 8 \times 10^{-7}$ has been set. If the decay were to occur via mixing, one would expect

$$B(K_S \to [\pi^+\pi^-\pi^0]|_{\text{CP}=-}) = \frac{\tau_S}{\tau_L}|\epsilon|^2 B(K_L \to \pi^+\pi^-\pi^0) \simeq 1.1 \times 10^{-9} \quad . \tag{58}$$

The decay $K_S \to [\pi^+\pi^-\pi^0]|_{\text{CP}=+}$ has been identified at CPLEAR.[51] This involves orbital angular momenta between the pions, as mentioned in Section 2 B.

3. One application of a ϕ factory, in which $K_S - K_L$ pairs are produced via the reaction $e^+e^- \to \phi \to K_S K_L$, is to select a K_L by means of some prominent decay mode and to look for (e.g.) the $K_S \to 3\pi^0$ decay. At least 10^{10} ϕ mesons are necessary to make a useful measurment; such numbers are envisioned for a machine currently under construction at Frascati near Rome.[53] The absence of $K_L K_L$ and $K_S K_S$ pairs in such a reaction follows from Bose statistics since the ϕ has $J = 1$.
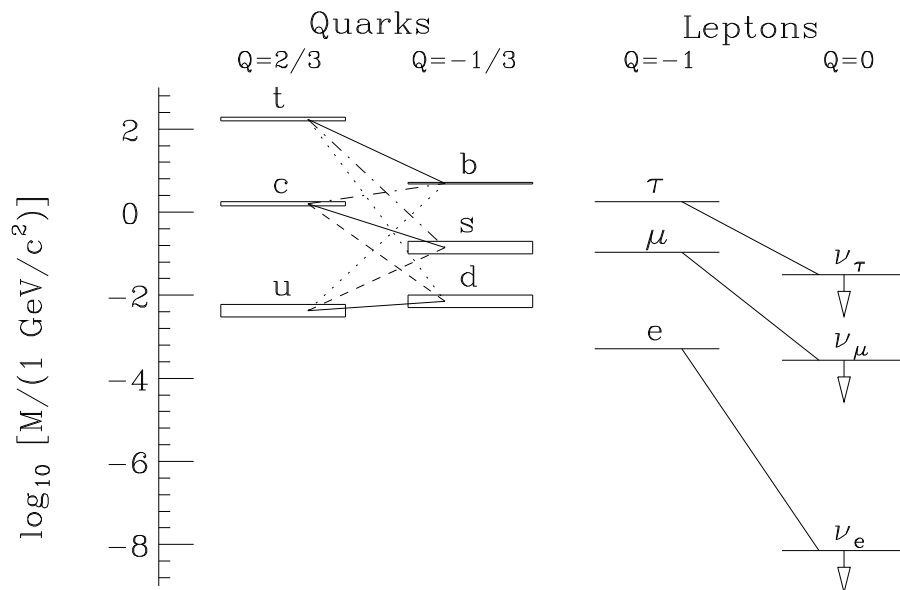
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Fig. 1. Patterns of charge-changing weak transitions among quarks and leptons. Direct evidence for ν_τ does not yet exist. The strongest inter-quark transitions correspond to the solid lines, with dashed, dot-dashed, and dotted lines corresponding to successively weaker transitions.

3. PHYSICS OF THE CKM MATRIX

A.   Quark masses and transitions

Our understanding of the weak interactions has undergone tremendous progress in the past century. It is almost 100 years since beta-decay electrons were first identified by J. J. Thomson in 1897. Fermi first formulated the theory of beta-decay as a charge-changing process. The space-time properties of such processes were finally settled in terms of $V - A$ interaction[4] nearly a quarter of a century later. We now understand the charge-changing weak interactions as part of a unified structure,[54] encompassing also electromagnetism and charge-*preserving* weak interactions.

Both leptons and quarks participate in the charge-changing weak interactions. The patterns, however, appear to be radically different.

At present, for lack of better information, we view each charged lepton as undergoing charge-changing transitions to or from its own neutrino, as illustrated on the right-hand side of Fig. 1. On the other hand, the quarks as shown on the left-hand side of Fig. 1, participate in a rich pattern of charge-changing transitions. This pattern is summarized in a 3×3 unitary matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix.[11,13]

We summarize the approximate relative strengths of the charge-changing weak transitions in Table 1. The relative phases of these amplitudes are also of importance. We shall describe how all this information is obtained after a discussion of the way in which the CKM matrix arises.

B.   Origin of the CKM matrix

The electroweak Lagrangian, before electroweak symmetry breaking, may be written in flavor-diagonal form as

$$\mathcal{L}_{\text{int}} = -\frac{g}{\sqrt{2}}[\overline{U'}_L \gamma^\mu W_\mu^{(+)} D'_L + H.c.] \quad , \tag{59}$$

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Table 1. Relative strengths of charge-changing weak transitions.

Relative amplitude	Transition	Source of information (example)
~ 1	$u \leftrightarrow d$	Nuclear β-decay
~ 1	$c \leftrightarrow s$	Charmed particle decays
~ 0.22	$u \leftrightarrow s$	Strange particle decays
~ 0.22	$c \leftrightarrow d$	Neutrino prod. of charm
~ 0.04	$c \leftrightarrow b$	b decays
~ 0.003	$u \leftrightarrow b$	Charmless b decays
~ 1	$t \leftrightarrow b$	Only indirect evidence
~ 0.04	$t \leftrightarrow s$	Only indirect evidence
~ 0.01	$t \leftrightarrow d$	Only indirect evidence

where $U' \equiv (u', c', t')$ and $D' \equiv (d', s', b')$ are column vectors decribing *weak eigenstates*. Here g is the weak $SU(2)_L$ coupling constant, and $\psi_L \equiv (1 - \gamma_5)\psi/2$ is the left-handed projection of the fermion field $\psi = U$ or D.

Quark mixings arise because mass terms in the Lagrangian are permitted to connect weak eigenstates with one another. Thus, the matrices $\mathcal{M}_{U,\,D}$ in

$$\mathcal{L}_m = -[\overline{U'}_R \mathcal{M}_U U'_L + \overline{D'}_R \mathcal{M}_D D'_L + H.c.] \tag{60}$$

may contain off-diagonal terms. One may diagonalize these matrices by separate unitary transformations on left-handed and right-handed quark fields:

$$R_Q^+ \mathcal{M}_Q L_Q = L_Q^+ \mathcal{M}_Q^+ R_Q = \Lambda_Q \quad . \tag{61}$$

where

$$Q'_L = L_Q Q_L; \quad Q'_R = R_Q Q_R \quad (Q = U, D) \quad . \tag{62}$$

Using the relation between weak eigenstates and mass eigenstates: $U'_L = L_U U_L$, $D'_L = L_D D_L$, we find

$$\mathcal{L}_{\text{int}} = -\frac{g}{\sqrt{2}} [\overline{U}_L \gamma^\mu W_\mu V D_L + H.c.] \quad , \tag{63}$$

where $U \equiv (u, c, t)$ and $D \equiv (d, s, b)$ are the mass eigenstates, and $V \equiv L_U^+ L_D$. The matrix V is just the Cabibbo-Kobayashi-Maskawa matrix. By construction, it is unitary: $V^+ V = V V^+ = 1$. It carries no information about R_U or R_D. More information would be forthcoming from interactions sensitive to right-handed quarks or from a genuine theory of quark masses.

C.   Parameter counting

For n u-type quarks and n d-type quarks, V is $n \times n$ and unitary. An arbitrary $n \times n$ matrix has $2n^2$ real parameters, but unitarity ($V^+ V = 1$) provides n^2 constraints, so only n^2 real parameters remain. We may remove $2n - 1$ of these by appropriate redefinitions of relative quark phases. The number of remaining parameters is then $n^2 - (2n - 1) = (n - 1)^2$. Of these, $n(n - 1)/2$ (the number of independent rotations in n dimensions) correspond to angles, while the rest, $(n - 1)(n - 2)/2$, correspond to phases. We summarize these results in Table 2.

For $n = 2$, we have one angle and no phases. The matrix V then can always be chosen as orthogonal.[11,12] For $n = 3$, we have three angles and one phase, which in general cannot be eliminated by arbitrary choices of phases in the quark fields. It was this phase that motivated Kobayashi and Maskawa[13] to introduce a third quark doublet. It provides a potential source of CP violation, serving

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Table 2. Parameters of KM matrices for n doublets of quarks.

	$n =$	2	3	4
Number of parameters	$(n-1)^2$	1	4	9
Number of angles	$n(n-1)/2$	1	3	6
Number of phases	$(n-1)(n-2)/2$	0	1	3

as the leading contender for the observed CP-violating effects in the kaon system and suggesting substantial CP asymmetries in the decays of mesons containing b quarks.

The CKM matrix V is then, explicitly,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad . \tag{64}$$

We now parametrize its elements.

It is convenient to choose quark phases[55] so that the n diagonal elements and the $n-1$ elements just above the diagonal are real and positive. Since all the angles θ_{ij} are small, the V_{cs} element in (64) is nearly real, so only a small change in quark phases is needed to accomplish this. The parametrization we shall employ is one suggested by Wolfenstein.[15]

The diagonal elements of V are nearly 1, while the dominant off-diagonal elements are $V_{us} \simeq -V_{cd} \equiv \lambda \simeq 0.22$. Thus to order λ^2, the upper 2×2 submatrix of V is already known from the four-quark pattern. The empirical observation that $V_{cb} \simeq 0.04$ allows one to express it as $A\lambda^2$, where $A = \mathcal{O}(1)$. Unitarity then requires $V_{ts} \simeq -A\lambda^2$ as long as V_{td} and V_{ub} are small enough (which they are). Finally, V_{ub} appears to be of order $A\lambda^3 \times \mathcal{O}(1)$. Here one must allow for a phase, so one must write $V_{ub} = A\lambda^3(\rho - i\eta)$. Finally, unitarity specifies uniquely the form $V_{td} = A\lambda^3(1 - \rho - i\eta)$. To summarize, the CKM matrix may be written

$$V \approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} \quad . \tag{65}$$

We shall anticipate further results to note that $V_{cb} = 0.038 \pm 0.003$. This enables us to write $A = 0.79 \pm 0.06$. The measurement of semileptonic charmless B decays gives $|V_{ub}/V_{cb}|$ in the range from 0.06 to 0.10, where most of the uncertainty is associated with the spread in models for the lepton spectra. Taking 0.08 ± 0.02 for this ratio, we find that the corresponding constraint on ρ and η is $(\rho^2 + \eta^2)^{1/2} = 0.36 \pm 0.09$. The main indeterminacy in the CKM matrix concerns the magnitude of V_{td}, for which only indirect evidence exists.

The form (65) is only correct to order λ^3 in the matrix elements. For certain purposes it may be necessary to exhibit corrections of higher order to the elements. This can be done using the unitarity of the matrix.

Unitarity implies that $V_{ij}^* V_{ik} = \delta_{jk}$ and $V_{ij}^* V_{kj} = \delta_{ik}$, where summation over repeated indices is understood. For example, we have

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0 \quad . \tag{66}$$

Since $V_{ud}^* \approx 1$, $V_{us}^* \approx \lambda$, $V_{ts} \approx -A\lambda^2$, and $V_{tb} \approx 1$ we have $V_{td} + V_{ub}^* = A\lambda^3$, a useful relation expressing the least-known CKM elements in terms of relatively well-known parameters. This result can be visualized as a triangle[56] in the complex plane [Fig. 2(a)]. In this figure the angles α, β, and γ are defined as in the review by Nir and Quinn.[57]
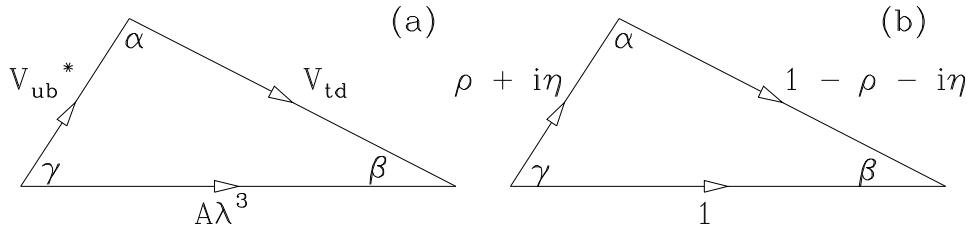
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Fig. 2. Unitarity triangle for CKM elements. (a) The relation (66) in the complex plane; (b) Eq. (66) divided by the normalizing factor $A\lambda^3$.

Dividing (66) by $A\lambda^3$, since $V_{ub}^*/A\lambda^3 = \rho + i\eta$, $V_{td}/A\lambda^3 = 1 - \rho - i\eta$, one obtains a triangle of the form shown in Fig. 2(b). The value of $V_{ub}^*/A\lambda^3$ may then be depicted as a point in the (ρ, η) plane. The major ambiguity which still remains in the determination of the CKM matrix elements concerns the shape of the unitarity triangle. The answer depends on the magnitude of V_{td}. As we shall see, decays alone will not provide the answer. One resorts to indirect means, which involve loop diagrams.

D.   Direct measurements of elements

1. The matrix element $|V_{ud}|^2$ may be obtained by comparing the strengths of certain beta-decay transitions involving vector transitions with that of muon decay. One can also measure the neutron decay rate (which involves both vector and axial vector transitions), and extract the vector coupling strength by finding g_A from decay asymmetries. This vector coupling strength may be compared with that obtained in muon decay to learn $|V_{ud}|^2$. Finally, one can study the decay $\pi^+ \to \pi^0 e^+ \nu_e$.

A nuclear beta-decay process typically involves unknown matrix elements of both vector and axial currents. In transitions between two 0^+ states which belong to the same isospin multiplet, however, the conserved vector current hypothesis[4,58] serves to normalize the vector current. These transitions are known as *superallowed Fermi transitions.*

In order to extract a fundamental coupling strength from beta-decay lifetimes one must apply a phase space and Coulomb correction factor, independent of the decaying nucleus, and further nuclear-dependent radiative corrections. The results for eight nuclei: ^{14}O, ^{26}Al, ^{34}Cl, ^{38}K, ^{42}Sc, ^{46}V, ^{50}Mn, and ^{54}Co lead to an average $|V_{ud}| = 0.9740 \pm 0.0006$. However, the possibility of systematic effects which become more important for high-Z nuclei have led Marciano[59] to place most reliance in the ^{14}O value, with the error reflecting systematic as well as statistical uncertainty: $|V_{ud}| = 0.9748 \pm 0.0010$.

The measurements of the neutron lifetime and g_A have now become precise enough that one can extract a value of V_{ud}:[24] $|V_{ud}| = 0.9804 \pm 0.0005$ (rad. corrs.) ± 0.0010 (τ_n) ± 0.0020 (g_A), a bit above the values implied by superallowed Fermi beta-decay transitions.

The decay rate $\Gamma(\pi^+ \to \pi^0 e^+ \nu_e)$ is governed by a form factor which is nearly at zero momentum transfer, and radiative corrections appear to be well in hand. From present measurements one obtains $|V_{ud}| = 0.968 \pm 0.018$.[60] Improvement by a factor of 8 would match the precision of the neutron experiments.

2. The matrix element V_{us} is probed by the semileptonic decays of strange particles. The vector current matrix elements provide the most reliable information, since they are affected only to second order by SU(3) symmetry-breaking effects.[61] Two main sources of information on the vector $\Delta S = 1$ (strangeness-changing) current are the decays $K \to \pi\ell\nu$ (known as $K_{\ell 3}$ decays) and the semileptonic decays of hyperons.[62] The result obtained from $K_{\ell 3}$ decays is $V_{us} = 0.2196 \pm 0.0023$, while that obtained in a recent fit to semileptonic hyperon decays is $|V_{us}| = 0.220 \pm 0.001 \pm 0.003$. In what follows we shall use an average of the K_{e3} and hyperon values:[59] $|V_{us}| = 0.220 \pm 0.002$.

3. The quantity $|V_{cd}|$ can be learned from deep inelastic neutrino excitation of charm via reactions such as $\nu_\mu d \to \mu^- c$ and $\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{c}$ and from semileptonic decays of charmed mesons to

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nonstrange final states. Both are associated with some model-dependence. The average of results from deep inelastic scattering[63] is $|V_{cd}| = 0.205 \pm 0.011$. This value is compatible with the Cabibbo-Glashow-Iliopoulos-Maiani[11,12] four-quark picture, which would imply $V_{cd} = -V_{us}$. The semileptonic decays of charmed particles to nonstrange final states also lead to values compatible with this hypothesis.[64]

4. The matrix element V_{cs} is provided by neutrino interactions and charmed particle semileptonic decays. The neutrino reactions $\nu_\mu s \to \mu^- c$ and $\bar{\nu}_\mu \bar{s} \to \mu^+ \bar{c}$ both must proceed off strange quarks or antiquarks in the nucleon sea. Some uncertainty is associated with modeling this component of the proton. One then looks for semileptonic decays of the c or $\bar{c}$, leading (as above) to dimuon events. This method[63] leads to a lower bound $|V_{cs}| \geq 0.59$. The semileptonic charmed particle decays providing information on V_{cs} are $D^0 \to K^- e^+ \nu_e$ and $D^+ \to \overline{K}^0 e^+ \nu_e$. With assumptions about the form factor,[65] one finds $|V_{cs}| = 1.07 \pm 0.16$.

5. The measurement of $|V_{cb}|$ constituted the first evidence for a structure of the $\Delta Q = 1$ weak transitions extending beyond the four-quark picture. It is now known that the dominant decays of b quarks involve charmed quarks in the final state, so that $|V_{cb}| > |V_{ub}|$, but the rather small value of $|V_{cb}|$ proved to be a surprise.

The underlying physics in extracting V_{cb} from data involves measurement of the rate for the semileptonic decay of B mesons (mesons containing the b quark), and careful accounting of strong interaction effects relating this process to the underlying $b \to c\ell\nu$ transition strength. A free-quark estimate[24] suffices to illustrate the main features. For the decay of a free b quark to a light quark q and an additional fermion pair $A\overline{B}$, the predicted rate is

$$\Gamma(b \to q\overline{A}B) = \frac{G_F^2 m_b^5 N_c}{192\pi^3} |V_{qb}|^2 \Phi\left(\frac{m_q}{m_b}, \frac{m_A}{m_b}, \frac{m_B}{m_b}\right) \quad , \tag{67}$$

where $N_c = 1$ when $\overline{A}B$ is a lepton pair and 3 when it is a quark pair (in which case one should also multiply the result by $|V_{AB}|^2$). The kinematic factor Φ is[66]

$$\Phi(x, z, y) = 12 \int_{(x+y)^2}^{(1-z)^2} \frac{ds}{s}(s - x^2 - y^2)(1 + z^2 - s)$$

$$\times \left\{[s - (x-y)^2][s - (x+y)^2][(1+z)^2 - s][(1-z)^2 - s]\right\}^{1/2} \quad . \tag{68}$$

Specific limits include $\Phi(0, 0, 0) = 1$ and

$$\Phi(x, 0, 0) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x \quad . \tag{69}$$

For $q = c$ and $\overline{A}B = \bar{\nu}_\ell \ell$ the partial rate is (neglecting m_ℓ)

$$\Gamma(b \to c\bar{\nu}_\ell \ell^-) = 7.14 \times 10^{-11} \text{ GeV} \left(\frac{m_b}{5 \text{ GeV}}\right)^5 |V_{cb}|^2 \Phi\left(\frac{m_c}{m_b}, 0, 0\right) \quad . \tag{70}$$

This is to be compared with the measured semileptonic b decay rate

$$\Gamma(B \to \text{charm} + \bar{\nu}_\ell + \ell^-) = \frac{\hbar B(B \to \text{charm} + \bar{\nu}_\ell + \ell^-)}{\tau_B} \tag{71}$$

For $B(B \to \text{charm} + \bar{\nu}_\ell + \ell^-) = 10.5\%$ and $\tau_B = 1.49$ ps it was then concluded,[24] based on the ranges $m_b = 5.0 \pm 0.3 \text{ GeV}/c^2$ and $m_b - m_c = 3.37 \pm 0.03 \text{ GeV}/c^2$ in typical descriptions of hadron spectra, that

$$|V_{cb}| = 0.038 \pm 0.003 \quad ; \quad A \equiv |V_{cb}|/|V_{us}|^2 = 0.785 \pm 0.062 \quad . \tag{72}$$

More recent data[67] imply a slightly longer B meson lifetime (the average of $\tau(B^+)$ and $\tau(B^0)$ is 1.63 ± 0.05 ps), and an inclusive semileptonic branching ratio of $10.98 \pm 0.28\%$ (with some additional

systematic error associated with choice of model). The result (72) nonetheless continues to represent the range of values obtained under various assumptions[68] about the way in which free quarks are incorporated into hadrons, and is consistent with a recent determination[69] with slightly smaller quoted errors.

6. The magnitude of V_{ub} may be estimated by studying the spectra of charged leptons in the decays $b \rightarrow q\bar{\nu}_\ell \ell^-$, which are sensitive to the relative contributions of $b \rightarrow u\bar{\nu}_e e^-$ and $b \rightarrow c\bar{\nu}_e e^-$ and hence to $|V_{ub}/V_{cb}|$. Defining $x \equiv 2E_e/m_b$, where E_e is the electron energy in the b rest frame, $\Gamma_0 \equiv G_F^2 m_b^5/(192\pi^3)$, and $\zeta \equiv m_q^2/m_b^2$, we have[70]

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx} = 2x^2|V_{qb}|^2 \left(\frac{1-\zeta-x}{1-x}\right)^2 \left[1-\zeta-x+\frac{2-x}{1-x}(1+2\zeta-x)\right] \quad . \tag{73}$$

When $\zeta = 0$ one obtains the Michel spectrum $\sim 2x^2(3-2x)$, which attains its maximum at $x = 1$. When $\zeta \neq 0$ the spectrum falls to zero at the kinematic limit $x = 1 - \zeta$. Corrections to the shape from perturbative and nonperturbative strong interactions are important,[71] but the main points already can be visualized without them.

The integrals over the spectra for $b \rightarrow u\ell^-\bar{\nu}_\ell$ and $b \rightarrow c\ell^-\bar{\nu}_\ell$ lead to about a factor of two kinematic suppression of $b \rightarrow c\ell^-\bar{\nu}_\ell$ relative to $b \rightarrow u\ell^-\bar{\nu}_\ell$. The spectrum of leptons emitted in semileptonic B decays is consistent on the whole with the $b \rightarrow c\ell\bar{\nu}_\ell$ prediction after corrections for QCD effects and detector resolution are taken into account. The decay $b \rightarrow c\ell\bar{\nu}_\ell$ is far more prevalent than $b \rightarrow u\ell\bar{\nu}_\ell$. However, by studying leptons with energies beyond the endpoint for $b \rightarrow c\ell\bar{\nu}_\ell$, both ARGUS[72] and CLEO[73] have concluded that $b \rightarrow u\ell\bar{\nu}_\ell$ does occur, at a rate about 2% (ARGUS) or 1% (CLEO) of that for $b \rightarrow c\ell\bar{\nu}_\ell$. When combined with the spread in various models[74] for deviations from the free-quark predictions, these measurements imply that

$$|V_{ub}/V_{cb}| = 0.08 \pm 0.02 \quad ; \quad (\rho^2 + \eta^2)^{1/2} = 0.36 \pm 0.09 \quad . \tag{74}$$

The use of exclusive decays such as $B \rightarrow (\pi, \rho, \text{or } \omega)\ell\nu$ may lead in the future to more restrictive bounds on $|V_{ub}|$. Some recent progress in identifying $B \rightarrow \pi\ell\nu$ decays has been reported by the CLEO Collaboration.[75]

A measurement of the branching ratio for $B \rightarrow \tau\nu$ or $B \rightarrow \mu\nu$, though challenging, will be helpful in extracting the product $f_B|V_{ub}|$, where the B decay constant f_B is defined by

$$\langle 0|J^\mu|B(q)\rangle = iq^\mu f_B \quad . \tag{75}$$

The predicted decay rates are

$$\Gamma(B^+ \rightarrow l^+\nu_l) = \frac{G_F^2 f_B^2 m_l^2 m_B}{8\pi} \left[1 - \frac{m_l^2}{m_B^2}\right]^2 |V_{ub}|^2 \quad . \tag{76}$$

Here we have chosen a normalization such that the analogous decay constant for pions[59] is $f_\pi = 132$ MeV. The expected branching ratios are

$$B(B^+ \rightarrow \mu^+\nu_\mu) = (1.1 \times 10^{-7})(f_B/f_\pi)^2|V_{ub}/0.003|^2 \quad ;$$

$$B(B^+ \rightarrow \tau^+\nu_\tau) = (2.5 \times 10^{-5})(f_B/f_\pi)^2|V_{ub}/0.003|^2 \quad . \tag{77}$$

Recent estimates[76,77] suggest $f_B \approx (1.4 \pm 0.2)f_\pi$. The hadronic form factors for decays like $B \rightarrow \pi l\nu$, $B \rightarrow \rho l\nu$, or $B \rightarrow \omega l\nu$ may be related to those of $D \rightarrow \pi l\nu$, $\rho l\nu$ or $\omega l\nu$ using heavy-quark symmetries,[68] making it possible to learn $|V_{ub}|$ from exclusive processes once the Dalitz plots of these D decays have been studied.

7. Elements involving the top quark have not yet been measured directly. From the unitarity of the CKM matrix, we expect $|V_{tb}| \simeq 1, |V_{ts}| \simeq 0.04$, and $|V_{td}| \simeq 0.005$ to 0.012. For $m_t > M_W + m_b$, t
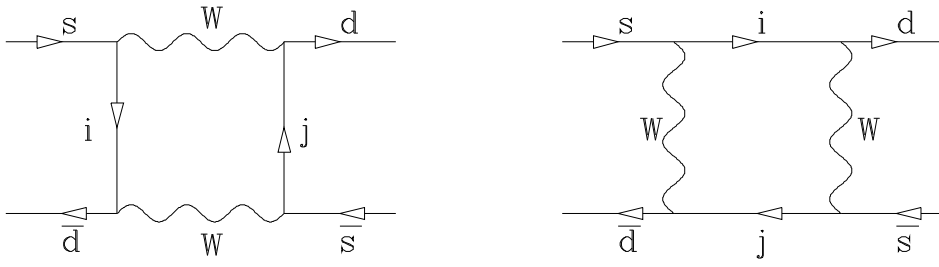
Fig. 3. Box diagrams for mixing of a neutral kaon and its antiparticle.

will decay to $W + (s$ or $d)$ with a branching ratio less than 2×10^{-3}, with the dominant decay being to $W + b$. The decay $t \to Wb$ is expected to have a partial width of 1.8 ± 0.4 GeV for $m_t = 180 \pm 12$ GeV/c^2 and $|V_{tb}| \approx 1$.

We shall now discuss indirect measurements of $|V_{ts}|$ and $|V_{td}|$ by means of their contributions to box diagrams.

E. Box diagrams

Indirect information on the CKM matrix is provided by $B^0 - \bar{B}^0$ mixing and CP-violating $K^0 - \bar{K}^0$ mixing, through the contributions of box diagrams involving two charged W bosons and two quarks of charge 2/3 (u, c, t) on the intermediate lines. These calculations have acquired new precision as a result of evidence for the top quark with a mass of $m_t = 180 \pm 12$ GeV/c^2, where we have averaged values of $176 \pm 8 \pm 10$ GeV/c^2 from CDF[78] and $199\,^{+19}_{-21} \pm 22$ GeV/c^2 from D0.[79]

Details of calculations of box diagrams have been given in several places,[6,23,80] so we shall give only the main results. A key feature is the cancellation of the highest powers of momenta in the loop integrals as a result of the unitarity of the CKM matrix: $V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0$ or $V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$. An approximation whose validity must be checked by explicit calculation of hadronic matrix elements (e.g., in lattice gauge theory) is the assumption of "vacuum saturation," in which one takes the matrix element of the box diagram between K^0 and $\bar{K}^0$ by inserting the vacuum in all possible ways. One writes[6]

$$\langle K^0 | (\bar{d}La\gamma^\mu s_L^a)(\bar{d}Lb\gamma_\mu s_L^b) | \bar{K}^0 \rangle = -2m_K^2 f_K^2 B_K / 3 \quad , \tag{78}$$

where a and b are color indices, $f_K = 161$ MeV is the kaon decay constant, and $B_K = 1$ corresponds to the assumption of vacuum saturation.

F. CP-violating mixing of K mesons

A mass term in the Lagrangian is of the form $\mathcal{L}_m = -m^2 \phi_K^\dagger \phi_K$ where ϕ_K denotes the kaon field. To make contact with what is calculated from Feynman rules, we note that the electromagnetic interaction term in the Lagrangian, $\mathcal{L}_{\rm int}^{\rm e.m.} = -e\bar{\psi}\gamma^\mu A_\mu \psi$, corresponds to the Feynman rule $-ie\gamma_\mu$. Thus, if we denote by $A_{\rm eff}$ the amplitude calculated according to Feynman rules for $K - \bar{K}$ mixing, we have the correspondence

$$\frac{1}{i}\langle K^0 | A_{\rm eff} | \bar{K}^0 \rangle = \delta m^2 = 2m_K \mathcal{M}_{12} \quad . \tag{79}$$

The loop-diagram contribution to $A_{\rm eff}$ from the exchange of a pair of W bosons is illustrated in Fig. 3. Here $i, j = u, c, t$ denote the quarks in the intermediate states. One finds in the limit in which all quark masses are small compared to M_W that the matrix element between a K^0 and a $\bar{K}^0$ is

$$M_{12} = \frac{-G_F^2 m_K f_K^2 B_K}{12\pi^2} \left[m_c^2 \xi_c^2 + m_t^2 \xi_t^2 + 2m_c^2 \xi_c \xi_t \ln \frac{m_t^2}{m_c^2} \right] \quad , \tag{80}$$

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where $\xi_i \equiv V_{id}^* V_{is}$. The first, second, and third terms correspond to loop diagrams with two charmed quarks, two top quarks, and one of each, respectively.

The top quark mass appears not to be small in comparison with M_W. For this case, one must take into account the $p_\mu p_\nu / M_W^2$ terms in the W propagator.[80]

We shall use the above result (and its generalization for heavy top) only to calculate the *imaginary* part of M_{12}, which affects CP violation. The imaginary part is dominated by large momenta in the loop graph, and thus is an effect of short-distance physics. The *real* part, on the other hand, is dominated by the contribution of the charmed quark and hence of much lower momenta, and a short-distance calculation is probably unreliable. The loop diagrams of Fig. 3 do not contribute *too large* a mass difference between K_L and K_S. This result was one of the bases for the conclusion[14] that the charmed quark mass could not be more than a couple of GeV/c^2. A quantitative calculation of the real part of M_{12} involves contributions from π^0, η, η', and other single-particle states; a significant contribution of the 2π intermediate state,[81] as well as contributions from other low-mass states.

The box diagram's contributions to the parameter ϵ, according to the discussion in Sec. 2, may be evaluated from

$$\epsilon \simeq \frac{i \, \text{Im} \, M_{12}}{\mu_S - \mu_L} \simeq \frac{-\text{Im} \, M_{12} e^{i\pi/4}}{\sqrt{2}\Delta m} \quad . \tag{81}$$

The generalization of Eq. (80) to the case of general quark masses[80] then leads to

$$|\epsilon| \simeq \frac{G_F^2 m_K f_K^2 B_K M_W^2}{\sqrt{2}(12\pi^2)\Delta m}[\eta_1 S(x_c)\text{Im} \, \xi_c^2 + \eta_2 S(x_t)\text{Im} \, \xi_t^2 + 2\eta_3 S(x_c, x_t)\text{Im} \, \xi_c\xi_t] \quad . \tag{82}$$

The factors $\eta_1 = 0.85$, $\eta_2 = 0.61$, $\eta_3 = 0.36$ are QCD corrections,[82] while $x_i \equiv m_i^2/M_W^2$. The functions $S(x)$ and $S(x, y)$ are

$$S(x) \equiv \frac{x}{4}\left[1 + \frac{3 - 9x}{(x-1)^2} + \frac{6x^2 \ln x}{(x-1)^3}\right] \quad ; \tag{83}$$

$$S(x, y) \equiv xy\left\{\left[\frac{1}{4} + \frac{3}{2(1-y)} - \frac{3}{4(1-y)^2}\right]\frac{\ln y}{y - x} + (y \leftrightarrow x) - \frac{3}{4(1-x)(1-y)}\right\} \quad . \tag{84}$$

To evaluate the required imaginary parts of the ξ_i^2 or $\xi_i\xi_j$, we must use a slightly better approximation to the CKM matrix than was introduced in Sec. 3 C. The application of the unitarity relation to the first and second rows of the matrix tells us, in fact, that a more precise expression for V_{cd} is $V_{cd} = -\lambda - A^2\lambda^5(\rho + i\eta)$. We then find:

$$\text{Im} \, \xi_c^2 = 2\text{Re} \, \xi_c\text{Im} \, \xi_c = -2A^2\lambda^6\eta \quad ;$$

$$\text{Im} \, \xi_c\xi_t \simeq \text{Re} \, \xi_c\text{Im} \, \xi_t = A^2\lambda^6\eta \quad ;$$

$$\text{Im} \, \xi_t^2 = 2\text{Re} \, \xi_t\text{Im} \, \xi_t = 2A^2\lambda^6\eta[A^2\lambda^4(1-\rho)] \quad . \tag{85}$$

Eq. (82) may then be rewritten as[83]

$$|\epsilon| = 4.33A^2 B_K\eta[\eta_3 S(x_c, x_t) - \eta_1 S(x_c) + \eta_2 A^2\lambda^4(1-\rho)S(x_t)] \quad . \tag{86}$$

Using the experimental value[31] of $|\epsilon| = (2.26 \pm 0.02) \times 10^{-3}$, the value $B_K = 0.8 \pm 0.2$,[84] and the top quark mass $m_t = 180 \pm 12$ GeV/c^2, we find that CP-violating $K - \bar{K}$ mixing leads to the constraint

$$\eta(1 + 0.35 - \rho) = 0.48 \pm 0.20 \quad , \tag{87}$$

where the first term in parentheses corresponds to the loop diagram with two top quarks, and the second corresponds to the additional contribution of charmed quarks. The major source of error on the right-hand side is the uncertainty in the parameter $A \equiv V_{cb}/\lambda^2$.
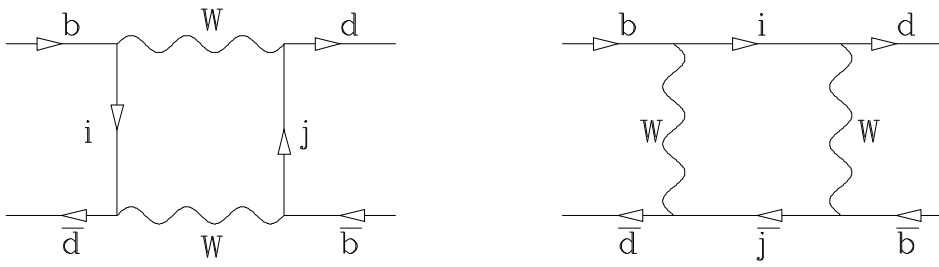
Fig. 4. Box diagrams for mixing of B^0 and $\bar{B}^0$.

F. Mixing of B mesons

The original evidence for $B^0 - \bar{B}^0$ mixing came from the presence of "wrong-sign" leptons in B meson semileptonic decays.[85] More recently, the time-dependence of $B^0 - \bar{B}^0$ oscillations has been traced by several groups.[67]

The splitting Δm_B between mass eigenstates is proportional to $f_B^2 m_t^2 |V_{td}|^2$ times a slowly varying function of m_t. Here f_B is the decay constant of the B meson [defined in Eq. (75)]. The contributions of lighter quarks in the box diagrams, while necessary to cut off the high-energy behavior of the loop integrals, are numerically insignificant.

The CKM element $|V_{td}|$ is proportional to $|1 - \rho - i\eta|$. Thus, exact knowledge of Δm_B, f_B and m_t would specify a circular arc in the (ρ, η) plane with center (1,0). Errors on all these quantities spread this arc out into a band.

A general parametrization of the mass matrix for states of two neutral mesons which can mix with one another can be written by transcribing the results of Sec. 2 as

$$\mathcal{M} = \begin{bmatrix} (\mu_L + \mu_S)/2 & p(\mu_S - \mu_L)/2q \\ q(\mu_S - \mu_L)/2p & (\mu_L + \mu_S)/2 \end{bmatrix} \quad . \tag{88}$$

Now, very few intermediate states are common to both B^0 and $\bar{B}^0$ decays, in contrast to the kaon case for which the $\pi\pi$ state is dominant. The B^0 (with a $\bar{b}$ quark) decays mostly to states with a $\bar{c}$ (such as $\bar{D}\pi$), while the $\bar{B}^0$ (with a b quark) decays mostly to states with a c (such as $D\pi$). As a result, the elements Γ_{12} and Γ_{21} are negligible in comparison with M_{12} and M_{21}, and $\Gamma_S \approx \Gamma_L$. Consequently, $\mu_S - \mu_L \approx m_S - m_L$, a real quantity. Then $\mu_S - \mu_L = 2\sqrt{\mathcal{M}_{12}\mathcal{M}_{21}}$ is nearly real, so that $p/q \approx (q/p)^*$, or

$$|p/q| \approx 1 \quad ; \quad |\mu_S - \mu_L| \approx 2|\mathcal{M}_{12}| \quad . \tag{89}$$

The relative magnitudes of the contributions for different $Q = 2/3$ quarks are easily estimated. Here, $\xi_c = V_{cd}^* V_{cb} \approx (-\lambda)(A\lambda^2)$ and $\xi_t = V_{td}^* V_{tb} \approx (A\lambda^3[1 - \rho + i\eta])(1)$ are comparable in magnitude, but $m_t \gg m_c$, so that the diagram with t quarks in both internal lines dominates the mass difference. Here, in contrast to the situation with kaons, a short-distance calculation of Δm is likely to be reliable. Furthermore, there are fewer shared intermediate states between B^0 and $\bar{B}^0$, so that mixing via such states is unlikely to be very important. Thus, the mixing should be given fairly reliably by the contributions shown in Fig. 4.

A crucial parameter probed by the graphs of Fig. 4, in addition to the top quark mass, is the KM element V_{td}. The expression for the ratio of mass shift to lifetime is

$$\frac{\Delta m}{\Gamma} = \frac{G_F^2}{6\pi^2} |V_{td}|^2 M_W^2 m_B f_B^2 B_B \eta_B \tau_B S\left(\frac{m_t^2}{M_W^2}\right) \quad . \tag{90}$$

Here f_B is defined such that the pion decay constant is 132 MeV. We shall take $f_B = 180 \pm 30$ MeV in accord with estimates[76,77] to be described in more detail in Sec. 4 A. B_B is a parameter describing
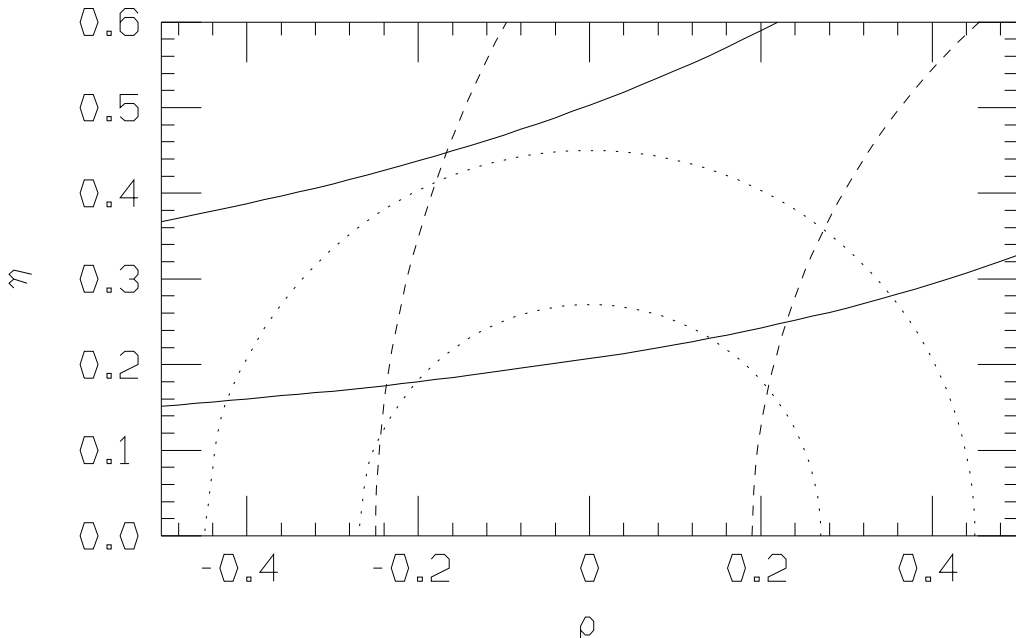
Fig. 5. Region in the (ρ, η) plane allowed by constraints on $|V_{ub}/V_{cb}|$ (dotted semicircles), $B^0 - \bar{B}^0$ mixing (dashed semicircles), and CP-violating $K - \bar{K}$ mixing (solid hyperbolae).

the degree of vacuum saturation for matrix elements analogous to (78) for the B mesons. We shall take $B_B = 1$; the calculation depends only on the product $f_B B_B^{1/2}$. The parameter η_B describes a QCD correction;[86] we take $\eta_B = 0.6 \pm 0.1$. The B^0 lifetime is taken[67] to be $\tau_B = 1.621 \pm 0.067$ ps. The function S is the one defined in Eq. (83).

Recent averages[67] for the $B^0 - \bar{B}^0$ mixing parameter $\Delta m_d = 0.462 \pm 0.026$ ps^{-1} and the B^0 lifetime $\tau(B^0) = 1.621 \pm 0.067$ ps can be combined to yield $\Delta m/\Gamma = 0.75 \pm 0.05$. If interpreted in terms of the box diagram for $b\bar{d} \to d\bar{b}$ (dominated by the top quark), this value leads to an estimate for $|V_{td}|$ reducing to $|1 - \rho - i\eta| = 1.03 \pm 0.22$. The dominant source of uncertainty in the right-hand side is the error on f_B. Prospects for reducing this error will be discussed in Sec. 4.

I. Allowed parameter space

Putting the constraints of the above discussion together, we find the allowed region of the (ρ, η) plane illustrated in Fig. 5. In each case the 1σ outer limits are quoted.

1. Charmless B decays lead to the constraint $(\rho^2 + \eta^2)^{1/2} = 0.36 \pm 0.09$ mentioned earlier, and hence to the dotted semicircles with center (0,0).

2. The parameter $|\epsilon|$ leads to the constraints denoted by the solid hyperbolae.

3. $B^0 - \bar{B}^0$ mixing leads to the constraints shown by the dashed circular arcs with center (0,1).

A large region centered about $\rho \simeq 0$, $\eta \simeq 0.35$ is permitted. Nonetheless, it could be that the CP violation seen in kaons is due to an entirely different source, perhaps a superweak mixing of K^0 and $\bar{K}^0$.[47] In that case one could probably still accommodate $\eta = 0$, and hence a real CKM matrix. In order to confirm the predicted nonzero value of η, we turn to other experimental possibilities.

4. IMPROVED TESTS OF THE STANDARD PICTURE

In this section we discuss several types of improved tests of the standard CKM picture of CP violation. We leave a discussion of the role of B mesons to Section 5.
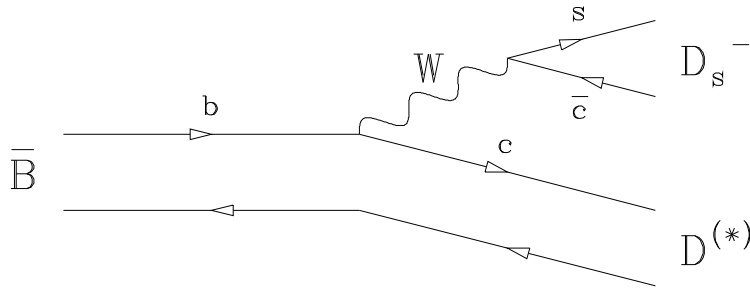
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Fig. 6. Application of factorization hypothesis to the decay $\bar{B} \to D_s^- D^{(*)}$. The weak current produces the D_s^-.

A.   Heavy meson decay constants

The decay constant f_B of the B^0 meson (the ground state of a b and a $\bar{d}$) affects the interpretation of $B^0 - \bar{B}^0$ mixing in terms of fundamental parameters of the CKM matrix (particularly $|V_{td}|$). Thus, it is important to know f_B better. The decay constants of other pseudoscalar mesons with one heavy quark and one light antiquark, such as $D_s = c\bar{s}$, $D^+ = c\bar{d}$, and $B_s = b\bar{s}$) are relevant auxiliary information. They allow one to check the validity of calculations, and are related to one another in the heavy-quark-symmetry limit by simple scaling relations.[68]

1. The D_s decay constant has recently been measured by several groups. Initial evidence for its value[87] was obtained by studying the decays $B \to D_s\bar{D}$ and assuming that the D_s was produced by the weak current. More recently, the decays $D_s \to \mu\nu$ and a few candidates for $D_s \to \tau\nu$ have been seen.[88] One obtains roughly $f_{D_s} = 300 \pm 50$ MeV.

2. The D^+ decay constant is expected on the basis of SU(3)-breaking estimates[76,77] to be 0.8 to 0.9 of f_{D_s}. Quark model arguments[77] favor the lower value, leading to the prediction $f_D = 240 \pm 40$ MeV. This is not far below the value obtained by the Mark III Collaboration[89] studying the decay $D \to \mu\nu$ at SPEAR: $f_D < 290$ MeV (90% c.l.).

3. The B^0 decay constant has been estimated both by means of lattice gauge theories[76] and via spin-dependent isospin splittings in D and B mesons,[77] which provide a value of the square of the wave function at zero interquark separation. Values in the range $f_B = 190 \pm 40$ MeV are obtained.

4. The B_s decay constant may be related to f_B by the SU(3) relation mentioned above for charmed mesons: $f_B/f_{B_s} = 0.8$ to 0.9, with 0.8 favored by quark models and some recent lattice models.[76,77] We then predict $f_{B_s} = 240 \pm 40$ MeV.

We now give some details of these results.

The factorization hypothesis as applied[87] to $\bar{B}$ decays to $D_s^- D^{(*)}$ is illustrated in Fig. 6. It is necessary to evaluate the form factors for emission of the current and to test factorization in other processes in order to utilize this method.

The emission of a current with 4-momentum q by a heavy quark Q_A in a transition to a heavy quark Q_B can be described in the limit of infinitely heavy quark masses by a universal form factor.[23,90] This quantity depends only on the invariant square of the difference $w = u - u'$ of the four-velocities $u \equiv p_A/m_A$ and $u' \equiv p_B/m_B$ of the initial and final heavy mesons. In terms of q^2 one has

$$w^2 = \frac{q^2 - (m_A - m_B)^2}{m_A m_B} = \frac{q^2 - q_{\max}^2}{m_A m_b} \quad . \tag{91}$$

For example, the form factors for $0^- \to 0^-$ and $0^- \to 1^-$ heavy-meson transitions are related to one another. (Here the symbol J^P denotes spin and parity.) Moreover, quark models or the heavy-quark-symmetry limit can be used to relate the production of a D_s by the current to production of a D_s^*. Thus, by measuring such processes as $B \to D^*\ell\nu$ at various values of momentum transfer, and using

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heavy-quark symmetry and the factorization hypothesis, one can predict the rate for all processes $\bar{B} \to D_s^{-(*)} D^{(*)}$ in which each final state meson is a pseudoscalar or a vector meson.

One can test factorization at $q^2 = m_\pi^2$ for pion emission, relying on the relation[91]

$$\frac{B(B \to D^* \pi)}{dB(B \to D^* \ell \nu)/dq^2}\big|_{q^2 = m_\pi^2} = 6\pi^2 f_\pi^2 |V_{ud}|^2 \quad . \tag{92}$$

The result[92] $f_\pi = 138 \pm 19$ MeV agrees with the actual value of 132 MeV. At $q^2 = m_{D_s}^2$ factorization may be more problematic, since final-state interactions are more likely to influence the result. Applying it nonetheless, the first estimates were obtained[92,93] for the D_s decay constant: $f_{D_s} = 276 \pm 69$ MeV.

The direct observations of D_s leptonic decays include the following.[88] The WA75 collaboration has seen $6 - 7$ $D_s \to \mu\nu$ events in emulsion and concludes that $f_{D_s} = 232 \pm 69$ MeV. A related experiment at Fermilab (the E653 Collaboration) also has events consistent with leptonic decays of D_s, but analysis is still in progress.[94] The CLEO Collaboration has a much larger statistical sample than WA75; the main errors arise from background subtraction and overall normalization (which relies on the $D_s \to \phi\pi$ branching ratio). Using several methods to estimate this branching ratio, Muheim and Stone estimate $f_{D_s} = 315 \pm 45$ MeV. A recent value from the BES Collaboration is $f_{D_s} = 434 \pm 160$ MeV (based on one candidate for $D_s \to \mu\nu$ and two for $D_s \to \tau\nu$). A reanalysis by F. Muheim of the Muheim-Stone result using the factorization hypothesis yields $f_{D_s} = 310 \pm 37$ MeV.

Quark models can provide estimates of decay constants and their ratios. In a non-relativistic model,[95] the decay constant f_M of a heavy meson $M = Q\bar{q}$ with mass M_M is related to the square of the $Q\bar{q}$ wave function at the origin by $f_M^2 = 12|\Psi(0)|^2/M_M$. The ratios of squares of wave functions can be estimated from strong hyperfine splittings between vector and pseudoscalar states, $\Delta M_{\text{hfs}} \propto |\Psi(0)|^2/m_Q m_q$. The equality of the $D_s^* - D_s$ and $D^* - D$ splittings then suggests that

$$f_D/f_{D_s} \simeq (m_d/m_s)^{1/2} \simeq 0.8 \simeq f_B/f_{B_s} \quad , \tag{93}$$

where we have assumed that similar dynamics govern the light quarks bound to charmed and b quarks.

An estimate of $|\Psi(0)|^2$ can be obtained using electromagnetic hyperfine splittings,[77] which are probed by comparing isospin splittings in vector and pseudoscalar mesons. Before corrections of order $1/m_Q$, the values $f_D^{(0)} = 290 \pm 15$ MeV, $f_B^{(0)} = 177 \pm 9$ MeV were obtained. With $f_M = f_M^{(0)}(1 - \Delta/m_M)$ ($M = D, B$), we use our value of f_D to estimate $\Delta/M_D = 0.20 \pm 0.11$, $\Delta/M_B = 0.07 \pm 0.04$, and hence $f_B = f_B^{(0)}(1 - \Delta/m_B) = 164 \pm 11$ MeV. Applying a QCD correction[96] of 1.10 to the ratio f_B/f_D, we finally estimate $f_B = 180 \pm 12$ MeV. [This is the basis of the central value taken in Section 3; the error of 30 MeV quoted there reflects our estimate of systematic uncertainties.] We also obtain $f_{B_s} = 225 \pm 15$ MeV from the ratio based on the quark model.

In a lattice gauge theory calculation, Bernard $et\ al.$[76] find

$$f_B = 187 \pm 10 \pm 34 \pm 15 \ \text{MeV} \quad ,$$

$$f_{B_s} = 207 \pm 9 \pm 34 \pm 22 \ \text{MeV} \quad ,$$

$$f_D = 208 \pm 9 \pm 35 \pm 12 \ \text{MeV} \quad ,$$

$$f_{D_s} = 230 \pm 7 \pm 30 \pm 18 \ \text{MeV} \quad , \tag{94}$$

where the first errors are statistical, the second are associated with fitting and lattice constant, and the third arise from scaling from the static ($m_Q = \infty$) limit. Duncan $et\ al.$[76] find

$$f_B = 188 \pm 23 \ (\text{stat}) \pm 15 \ (\text{syst}) \ {}^{+26}_{-0} \ (\text{extrap}) \pm 14 \ (\text{pert}) \ \text{MeV} \quad , \tag{95}$$

in accord with the above result. They obtain a ratio of strange to nonstrange B decay constants more in accord with our quark model estimates: $f_{B_s}/f_B = 1.22 \pm 0.04 \ (\text{stat}) \pm 0.02 \ (\text{syst})$.

B. Rates and ratios

Rates and ratios can constrain $|V_{ub}|$ and possibly $|V_{td}|$. The partial width $\Gamma(B \to \ell\nu)$ is proportional to $(f_B|V_{ub}|)^2$. The expected branching ratios are about $(1/2) \times 10^{-4}$ for $\tau\nu$ and 2×10^{-7} for $\mu\nu$, as mentioned in Sec. 3 D 6. The measurement of $B^0 - \bar{B}^0$ mixing can provide the combination $f_B|V_{td}|$. Dividing $f_B|V_{ub}|$ by $f_B|V_{td}|$, we can eliminate the decay constants, obtaining

$$\frac{|V_{ub}|}{|V_{td}|} \equiv r = \left[\frac{\rho^2 + \eta^2}{(1-\rho)^2 + \eta^2} \right]^{1/2} \tag{96}$$

with smaller errors than in $|V_{ub}|$ or $|V_{td}|$.[97] Contours of fixed r are circles in the (ρ, η) plane with radius $|r/(1-r^2)|$ and center $\rho_0 = -r^2/(1-r^2)$, $\eta_0 = 0$.

Another interesting ratio[98] is $\Gamma(B \to \rho\gamma)/\Gamma(B \to K^*\gamma)$, which, aside from phase space corrections, should be $|V_{td}/V_{ts}|^2 \simeq 1/20$. Soni, however,[99] has argued that there are likely to be long-distance corrections to this relation.

C. Rare kaon decays

A number of rare kaon decays can provide information on details of the CKM matrix and on fundamental aspects of CP violation. Other decays provide valuable auxiliary data. We give a brief sample of the information here, updating the experimental situation and referring the reader elsewhere[23,82,100,101] for more complete discussions of formalism.

1. The decay $K_L \to \gamma\gamma$ can proceed in a CP-conserving manner. It occurs with a branching ratio[31] of $(5.73 \pm 0.27) \times 10^{-4}$, or nearly 2/3 the branching ratio of the CP-violating $\pi^0\pi^0$ mode.[102] Calculations of the rate involve prominent roles for hadronic intermediate states such as π^0, η, and η' (whose contributions would sum to zero in the SU(3) limit), and thus involve long-distance strong-interaction physics.

The $K_L \to \gamma\gamma$ amplitude is important for several reasons: (a) It governs the main contribution to the decay $K_L \to \mu^+\mu^-$; residual contributions to this process can probe short-distance physics, as discussed below. (b) The related processes $K_L \to \gamma e^+ e^-$[103] and $K_L \to \gamma\mu^+\mu^-$[104] provide information on the behavior of the $K_L \to \gamma\gamma^*$ amplitude for a virtual photon γ^*, which is important in estimating certain long-distance contributions to $K_L \to \mu^+\mu^-$. (c) A potential background[105] to the process $K_L \to \pi^0 e^+ e^-$, to be described below, is the process $K_L \to \gamma_1 e^+ e^-$, where the positron or electron radiates a photon (γ_2).

2. The decay $K_S \to \gamma\gamma$, measured[102] to have a branching ratio of about 2×10^{-6}, is reliably predicted[106] from the imaginary part of its amplitude, which arises as a result of the sequence $K_S \to \pi^+\pi^- \to \gamma\gamma$. This lends some credence to similar attempts to estimate the rate for $K_L \to \gamma\gamma\pi^0$ (a process to be described below).

3. The decay $K_L \to \mu^+\mu^-$ is dominated by the two-photon intermediate state: $K_L \to \gamma\gamma \to \mu^+\mu^-$. The lower limit on the predicted branching ratio is given by the "unitarity bound" which is based on neglect of all other contributions (including off-shall photons). The most recent measurement of the rate[107] yields $B(K_L \to \mu^+\mu^-) = (6.86 \pm 0.37) \times 10^{-9}$, very close to the unitarity bound and limiting other contributions. Since loop diagrams involving the top quark are included in these contributions, an upper limit on $|V_{td}|$ ensues, entailing a lower bound on ρ of about -0.7.[107] As we see from Fig. 5, a more stringent lower bound of about -0.2 would begin to provide information more powerful than that supplied by $B^0 - \bar{B}^0$ mixing.

4. The decay $K^+ \to \pi^+ e^+ e^-$ proceeds primarily through a virtual photon via $K^+ \to \pi^+\gamma^* \to \pi^+ e^+ e^-$. The amplitude does not have a pole at $q^2 = 0$ since the real transition $K^+ \to \pi^+\gamma$ is forbidden by angular momentum conservation. (Both the K^+ and the π^+ are spinless.) The $K^+ \to \pi^+\gamma^*$ vertex has both short- and long-distance contributions. The experimental value of the branching ratio[108] is $B(K^+ \to \pi^+ e^+ e^-) = (2.75 \pm 0.23 \pm 0.13) \times 10^{-7}$ (under some assumptions about the form of the interaction).

If the process $K_S \to \pi^0\gamma^*$ has the same amplitude as $K^+ \to \pi^+\gamma^*$ (a possibility if both transitions are governed by the short-distance process $s \to d\gamma^*$), then one expects $B(K_S \to \pi^0 e^+ e^-) = (\tau_S/\tau_{K^+})B(K^+ \to \pi^+ e^+ e^-) \simeq 2 \times 10^{-9}$. This estimate becomes important in judging the potential for the decay $K_L \to \pi^0 e^+ e^-$ to yield new information on CP violation, as we shall see below.

5. The $K^+ \to \pi^+\nu\bar{\nu}$ decay rate rate is governed by loop diagrams involving the cooperation of charmed and top quark contributions. An approximate expression[110] for the branching ratio, summed over neutrino species, is

$$B(K^+ \to \pi^+\nu\bar{\nu}) \simeq 1.8 \times 10^{-6}|D(x_c) + D(x_t)\lambda^4(1 - \rho - i\eta)|^2 \quad , \tag{97}$$

where $x_i \equiv m_i^2/M_W^2$ and

$$D(x) \equiv \frac{1}{8}\left[1 + \frac{3}{(1-x)^2} - \left(\frac{4-x}{1-x}\right)^2\right]x\ln x + \frac{x}{4} - \frac{3}{4}\frac{x}{1-x} \quad . \tag{98}$$

The first and second terms in (97) are due to the contributions of the charm and top quarks.

Experimental observation of the $K^+ \to \pi^+\nu\bar{\nu}$ decay would lead to constraints involving circles in the (ρ, η) plane with centers at approximately $(1.4, 0)$.[109] The favored branching ratio is about 1.2×10^{-10}, give or take a factor of 2. Variables affecting the result include not only ρ and η (mainly ρ), but also the charmed quark mass and the value of A. A low value of the branching ratio within this range signifies $\rho > 0$, while a high value signifies $\rho < 0$. The present upper limit[109] is $B(K^+ \to \pi^+\nu\bar{\nu}) < 3 \times 10^{-9}$ (90% c.l.).

6. The decay $K_L \to \pi^0\gamma\gamma$ is a potential CP-conserving source of $K_L \to \pi^0 e^+ e^-$, a process expected to be mainly CP-violating. The key question is whether the two photons are produced in a $J = 2$ state, which can produce $e^+ e^-$ without a suppression factor which operates in the $J = 0$ state.

Two experiments have detected this mode,[111] yielding a branching ratio of $(1.71 \pm 0.28) \times 10^{-6}$ [for $m(\gamma\gamma)$ within certain limits].[31] Elementary calculations analogous to the rescattering model[106] which gave $K_S \to \gamma\gamma$ correctly or, equivalently, based on chiral perturbation theory, can be applied to the sequence of processes $K_L \to \pi^+\pi^-\pi^0$, $\pi^+\pi^- \to \gamma\gamma$, to estimate the contribution of the charged-pion pair to $K_L \to \pi^0\gamma\gamma$.[112] The resulting prediction of the rescattering model is that $B(K_L \to \pi^0\gamma\gamma) = 7.5 \times 10^{-7}$.

The contributions of the $\pi^+\pi^-$ intermediate state lead to a dominantly $J = 0$ $\gamma\gamma$ final state. Because of the chirality conservation in electromagnetic interactions, this final state is very inefficient in producing $e^+ e^-$ pairs. The corresponding $\gamma\gamma \to e^+ e^-$ rescattering prediction for $B(K_L \to \pi^0 e^+ e^-)$ is about 10^{-13}. This is far lower than the main contributions to $K_L \to \pi^0 e^+ e^-$, which we shall see are expected to be CP-violating. On the other hand, if there is any $J = 2$ component to the $\gamma\gamma$ final state in $K_L \to \pi^0\gamma\gamma$, it can produce $e^+ e^-$ much more readily, and the CP-conserving background to $K_L \to \pi^0 e^+ e^-$ becomes significant.

A $J_{\gamma\gamma} = 2$ contribution to $K_L \to \pi^0\gamma\gamma$ is provided by intermediate states of vector mesons V, e.g., via $K_L \to V\gamma \to \pi^0\gamma\gamma$, where the weak Hamiltonian can act at either the first or the second stage. Opinion seems divided,[113,114] but typical branching ratios predicted in such models are $B(K_L \to \pi^0\gamma\gamma) \simeq (1 - 3) \times 10^{-6}$, in somewhat better accord with the experimental value than the predictions of the $\pi^+\pi^-$ rescattering model or chiral perturbation theory. However, the models with large vector meson contributions to the decay $K_L \to \pi^0\gamma\gamma$ tend to involve much larger contributions for $m_{\gamma\gamma} \leq 2m_\pi$ than the rescattering or chiral pertubation theory models. Although the acceptance in the CERN experiment[111] is highly non-uniform, the authors are able to place an upper limit of about 12% on the amount of the decay $K_L \to \pi^0\gamma\gamma$ which comes for $m_{\gamma\gamma} \leq 2m_\pi$.

The CP-conserving process $K_L \to \pi^0\gamma\gamma \to \pi^0 e^+ e^-$ therefore is unlikely to be a major source of $K_L \to \pi^0 e^+ e^-$. This remote possibility could be laid to rest by more detailed studies of the decay $K_L \to \pi^0\gamma\gamma$.

7. The decay $K_L \to \pi^0 e^+ e^-$ is expected to be dominated by CP-violating contributions. Two types of such contributions are expected: "indirect," via the CP-positive K_1 component of $K_L =$
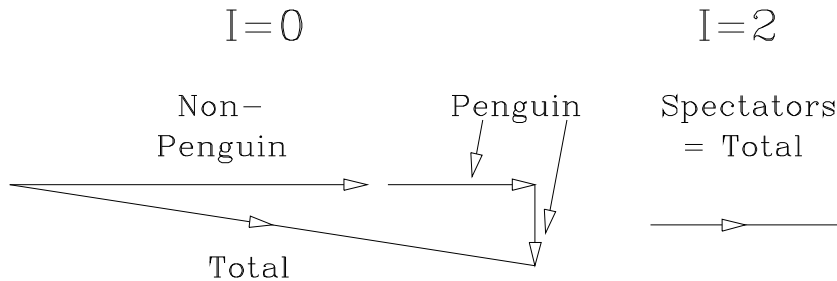
Fig. 7. Summary of contributions to $I = 0$ and $I = 2$ final states in $K \to \pi\pi$ decays.

$K_2 + \epsilon K_1$, and "direct," whose presence would be a detailed verification of the CKM theory of CP violation.

The indirect contribution is expected to lead to a branching ratio[100,101] $B^{\text{in}} \simeq 6 \times 10^{-12}$, if one uses the estimate quoted above that $B(K_S \to \pi^0 e^+ e^-) \simeq 2 \times 10^{-9}$. (One calculation[113] finds the $K_{S,L} \to \pi^0 e^+ e^-$ branching ratios an order of magnitude larger.) The phase of the indirect contribution is expected to be that of ϵ (i.e., about 45°) modulo π. The direct contribution should be proportional to $i\eta$, and should be comparable in magnitude to the indirect contribution in most estimates. One potential background[105] is the process $K_L \to \gamma_1 e^+ e^-$, where the positron or electron radiates a photon (γ_2). If $m(\gamma_1\gamma_2)$ is too close to m_{π^0}, this process can be confused with the signal.

The present 90% c.l. upper limit to $B(K_L \to \pi^0 e^+ e^-)$ is 1.8×10^{-9}, where results from several experiments have been combined.[115]

8. The decay $K_L \to \pi^0 \mu^+ \mu^-$ should have less background than $K_L \to \pi^0 e^+ e^-$ from photons radiated by the charged leptons, and should have a comparable rate (aside from phase space differences). The present 90% c.l. upper limit[116] is 5.1×10^{-9}.

9. The decay $K_L \to \pi^0 \nu\bar{\nu}$ should have a branching ratio of about $3 \times 10^{-10}\eta^2$ for $m_t = 180$ GeV.[100,101] It should have only a very small indirect contribution, and would be incontrovertible evidence for the CKM theory if observed at the predicted level. At present, experimental bounds[117] are $B(K_L \to \pi^0 \nu\bar{\nu}) < 5.8 \times 10^{-5}$ (90% c.l., summed over neutrino species).

D. ϵ'/ϵ

The ratio ϵ'/ϵ for kaons has long been viewed as one of the most promising ways to disprove a "superweak" theory of CP violation in neutral kaon decays.[47,101] We sketch the way in which ϵ' arises in the standard electroweak picture.[23,118]

Several types of weak decay amplitudes contribute to $K \to \pi\pi$.

1. The penguin graph, involving the transition $s \to d$ with emission of at least one gluon, leads only to an $I_{\pi\pi} = 0$ final state, since it can only change isospin by 1/2 unit. (Recall that the isospins of the two-pion $J = 0$ final states in kaon decays can be $I = 0$ and $I = 2$.) Since it involves three different generations of quarks as intermediate states, the penguin graph can give rise to a decay amplitude which is complex at the quark level. As we have seen from the discussion in Sec. 3, three generations are the minimum number for which this can occur.[13]

2. The W exchange diagram, describing the subprocess $s\bar{d} \to u\bar{u}$, also leads exclusively to an $I_{\pi\pi} = 0$ final state (since a $q\bar{q}$ state cannot have $I = 2$), and to a real amplitude A_0.

3. Free quark transitions of the form $s \to u\bar{u}d$, in which the $\bar{d}$ in the initial kaon acts as a spectator, can contribute a real part to both A_0 and A_2.

Simple diagrams in the complex plane (Fig. 7) show the difference between the total $I = 0$ and $I = 2$ amplitudes. As a result of the complex penguin diagram contribution, the two isospin amplitudes have a *nonzero relative phase*. Adopting the convention[40] in which A_0 is real, we may rotate the figures simultaneously by a small redefinition of the K and $\bar{K}$ fields, so that now it is A_2

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Table 3. Dependence of mixing parameter x_s on top quark mass and B_s decay constant.

m_t (GeV/c^2)	168	180	192
f_{B_s} (MeV)			
150	9	10	11
200	15	17	19
250	24	27	30

which has a small imaginary part. As mentioned in Sec. 2, the parameter ϵ' describing direct CP violation (that not due to mixing) in neutral kaon decays is then given by

$$\epsilon' = \frac{i}{\sqrt{2}} \frac{\text{Im } A_2}{A_0} e^{i(\delta_2 - \delta_0)} \quad . \tag{99}$$

The problem of calculating ϵ' then reduces to that of evaluating the relative contribution of penguin and non-penguin amplitudes to the *real* part of the $I = 0$ amplitude.[32] It is more straightforward to evaluate the phase of the penguin amplitude. The magnitude of the $I = 2$ amplitude could, in principle, be evaluated from first principles (though QCD corrections would be important), but in practice it may be simply taken from experiment.

Aadditional contributions and uncertainties are likely to decrease the overall magnitude of ϵ'/ϵ, and can even change its sign for top quarks heavier than about 200 GeV. The result should be corrected for isospin-breaking effects and for terms involving the replacement of the gluon emitted in the $s \to d$ transition by a photon or Z, whose coupling to the remaining quarks in the system is not isoscalar. Such diagrams can contribute directly to the $I = 2$ amplitude and constitute an important effect on ϵ'.[119]

The latest estimates[120] are equivalent (for a top mass of about 180 GeV/c^2) to $[\epsilon'/\epsilon]|_{\text{kaons}} = (6 \pm 3) \times 10^{-4}\eta$, with an additional factor of 2 uncertainty associated with hadronic matrix elements. The Fermilab E731 Collaboration's[45] value $\epsilon'/\epsilon = (7.4 \pm 6) \times 10^{-4}$ is consistent with η in the range (0.2 to 0.45) we have already specified. The CERN NA31 Collaboration's[46] value $\epsilon'/\epsilon = (23.0 \pm 6.5) \times 10^{-4}$ is higher than theoretical expectations. Both groups are preparing new experiments, which should begin taking data around 1996.

5. IMPORTANCE OF B HADRONS

A. Mixing of strange B mesons

$B_s - \bar{B}_s$ mixing probes the ratio $(\Delta m)|_{B_s}/(\Delta m)|_{B_d} = (f_{B_s}/f_{B_d})^2 (B_{B_s}/B_{B_d}) |V_{ts}/V_{td}|^2$, which should be a very large number (of order 20 or more). Thus, strange B's should undergo many particle-antiparticle oscillations before decaying.

The main uncertainty in an estimate of $x_s \equiv (\Delta m/\Gamma)_{B_s}$ is associated with f_{B_s}. The CKM elements $V_{ts} \simeq -0.04$ and $V_{tb} \simeq 1$ which govern the dominant (top quark) contribution to the mixing are known reasonably well. We show in Table 3 the dependence of x_s on f_{B_s} and m_t. To measure x_s, one must study the time-dependence of decays to specific final states and their charge-conjugates with resolution equal to a small fraction of $\tau(B_s) = 1.55 \pm 0.13$ ps.[67]

The estimate $f_B/f_{B_s} = 0.8 - 0.9$ and an experimental value for x_s would allow us to tell whether the unitarity triangle had non-zero area by specifying $|1 - \rho - i\eta|$.[27] Present bounds are not yet strong enough for this purpose. (See, e.g., Ref.,[121] for which the largest claimed lower bound is $x_s > 9$.) Assuming that $|V_{ub}/V_{cb}| > 0.06$, one must show $0.73 < |1 - \rho - i\eta| < 1.27$. Taking the B_s

and B_d lifetimes to be equal, and assuming $0.7 < x_d < 0.8$, this will be so if $16 < x_s < 34$. An "ideal" measurement would thus be $x_s = 25 \pm 3$.

The decay of a $\bar{B}_s = b\bar{s}$ meson via the quark subprocess $b(\bar{s}) \to c\bar{c}s(\bar{s})$ gives rise to neutral final states which turn out to be predominantly CP-even.[122] The mixing of $\bar{B}_s$ and B_s leads to eigenstates $B_s^{\pm}$ of even and odd CP; the predominance of CP-even final states formed of $c\bar{c}s\bar{s}$ means that the CP-even eigenstate will have a shorter lifetime. With $\Delta\Gamma(B_s) \equiv \Gamma(B_s^+) - \Gamma(B_s^-)$ and $\bar{\Gamma}(B_s) \equiv [\Gamma(B_s^+) + \Gamma(B_s^-)]/2$, Bigi *et al.*[123] estimate

$$\frac{\Delta\Gamma}{\bar{\Gamma}(B_s)} \simeq 0.18 \frac{f_{B_s}^2}{(200 \text{ MeV})^2} \quad , \tag{100}$$

possibly the largest lifetime difference in hadrons containing b quarks.

One could measure $\bar{\Gamma}(B_s)$ using semileptonic decays, while the decays to CP eigenstates could be measured by studying the correlations between the polarization states of the vector mesons in $B_s^{\pm} \to J/\psi K_S$. [For a similar method applied to decays of other pseudoscalar mesons see, e.g., Ref..[124]]

The ratio of the mass splitting to the width difference between CP eigenstates of strange B's is predicted to be large and independent of CKM matrix elements[125,126] (to lowest order, neglecting QCD corrections which may be appreciable):

$$\frac{\Delta m}{\Delta\Gamma} \simeq -\frac{2}{3\pi} \frac{m_t^2 h(m_t^2/M_W^2)}{m_b^2} \left(1 - \frac{8}{3}\frac{m_c^2}{m_b^2}\right)^{-1} \simeq 200 \ ! \tag{101}$$

Here $h(x)$ decreases monotonically from 1 at $x = 0$ to $1/4$ as $x \to \infty$; it is about 0.53 for the present value of m_t. If the mass difference $\Delta m/\bar{\Gamma} \approx 20$ turns out to be too large to measure, the width difference $\Delta\Gamma/\bar{\Gamma} \approx 1/10$ may be large enough to detect.

B. General time-dependent formalism for B mesons

1. Time dependences.[17] We may adapt the formalism introduced in Sec. 2 for eigenstates of the mass matrix to describe the time evolution of states which are initially B or $\bar{B}$ in terms of the evolution of mass eigenstates B_L and B_H. Here L and H stand for "light" and "heavy". The corresponding eigenvalues of the mass matrix are $\mu_{L,H} \equiv m_{L,H} - i\Gamma_{L,H}/2$. In terms of basis states $|B\rangle$ and $|\bar{B}\rangle$, we write

$$|B_L\rangle = p|B\rangle + q|\bar{B}\rangle \quad , \quad |B_H\rangle = p|B\rangle - q|\bar{B}\rangle \quad , \tag{102}$$

where we have assumed CPT invariance as in Sec. 2. It is convenient to define $\Delta\mu \equiv \mu_H - \mu_L = \Delta m - i\Delta\Gamma/2$; $\bar{\mu} \equiv (\mu_H + \mu_L)/2$; $m \equiv (m_H + m_L)/2$; and $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$. States B_L and B_H evolve as $|B_{L,H}\rangle \to |B_{L,H}\rangle e^{-i\mu_{L,H}t}$, so that

$$|B\rangle \to f_+(t)|B\rangle + \frac{q}{p}f_-(t)|\bar{B}\rangle \ ; \quad |\bar{B}\rangle \to \frac{p}{q}f_-(t)|B\rangle + f_+(t)|\bar{B}\rangle \quad , \tag{103}$$

where $f_{\pm}(t) \equiv (e^{-i\mu_L t} \pm e^{-i\mu_H t})/2$, or

$$f_+(t) = e^{-imt}e^{-\Gamma t/2}\cos(\Delta\mu t/2) \ ; \quad f_-(t) = e^{-imt}e^{-\Gamma t/2}i\sin(\Delta\mu t/2) \quad . \tag{104}$$

The time-integrated probabilities for detection of B or $\bar{B}$, given an initial $\bar{B}$, are

$$\int_0^{\infty} dt|f_{\pm}(t)|^2 = \frac{(\Delta m/\Gamma)^2 + \left\{ \begin{matrix} 2 \\ 0 \end{matrix} \right\}}{2 + 2(\Delta m/\Gamma)^2} \quad , \tag{105}$$

so (for $|p/q| = 1$, which we shall see is approximately true for B's) the ratio of "wrong-sign" to "right-sign" decays is equal to

$$\frac{\int_0^{\infty} |f_-(t)|^2 dt}{\int_0^{\infty} |f_+(t)|^2 dt} = \frac{(\Delta m/\Gamma)^2}{2 + (\Delta m/\Gamma)^2} \quad . \tag{106}$$

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For $\Delta m/\Gamma \simeq 0.75$, which is approximately the case for $B^0 - \bar{B}^0$ mixing, this ratio is about 0.22, while for $\Delta m/\Gamma \simeq 20$ (a typical value expected for $B_s - \bar{B}_s$ mixing, as noted in Sec. 5 A) the ratio (106) is 0.995.

2. CP-violating asymmetries for B's. We shall be interested in comparing rates for production of a final state $|f\rangle$ with those for the CP-conjugate final states $|\bar{f}\rangle \equiv CP|f\rangle$. Denote a state which is initially B^0 as $B^0_{\text{phys}}(t)$, and correspondingly for $\bar{B}^0$. These states evolve in time as described by Eq. (103). Then

$$\langle f|B^0_{\text{phys}}(t)\rangle = f_+(t)\langle f|B\rangle + \frac{q}{p}f_-(t)\langle f|\bar{B}\rangle \quad ,$$

$$\langle \bar{f}|\bar{B}^0_{\text{phys}}(t)\rangle = \frac{p}{q}f_-(t)\langle \bar{f}|B\rangle + f_+(t)\langle \bar{f}|\bar{B}\rangle \quad . \tag{107}$$

Now define

$$x \equiv \frac{\langle f|\bar{B}\rangle}{\langle f|B\rangle} \quad ; \quad \lambda_0 \equiv \frac{q}{p}x \quad ; \quad x \equiv \frac{\langle \bar{f}|\bar{B}\rangle}{\langle \bar{f}|B\rangle} \quad ; \quad \bar{\lambda}_0 \equiv \frac{p}{q}x \quad . \tag{108}$$

Then

$$\langle f|B^0_{\text{phys}}(t)\rangle = \langle f|B\rangle\{f_+(t) + \lambda_0 f_-(t)\} \quad ,$$

$$\langle \bar{f}|\bar{B}^0_{\text{phys}}(t)\rangle = \langle \bar{f}|\bar{B}\rangle\{f_+(t) + \bar{\lambda}_0 f_-(t)\} \quad . \tag{109}$$

Several simplifications often are possible. If each factor in x and $\bar{x}$ is dominated by a strong eigenchannel, final state phases cancel, and one can use the discussion of Sec. 2 E to show that $\bar{x} = x^*$. For B mesons, where the mixing is dominated by the box diagrams containing internal top quarks, one has

$$\frac{p}{q} \simeq \left(\frac{M_{12}}{M_{21}}\right)^{1/2} \simeq \frac{V_{td}^* V_{tb}}{V_{td} V_{tb}^*} \tag{110}$$

so that $|p/q| \simeq 1$. Combining this result with that for x, we find $\bar{\lambda}_0 = \lambda_0^*$.

The time-integrated asymmetry C_f for production of a final state f is defined as

$$C_f \equiv \frac{\Gamma(B^0_{\text{phys}} \to f) - \Gamma(\bar{B}^0_{\text{phys}} \to \bar{f})}{\Gamma(B^0_{\text{phys}} \to f) + \Gamma(\bar{B}^0_{\text{phys}} \to \bar{f})} \quad . \tag{111}$$

With $\bar{\lambda}_0 = \lambda_0^*$, and neglecting $\Delta\Gamma/\Gamma$ with respect to $\Delta m/\Gamma$, we find

$$C_f \equiv \frac{-2z \text{ Im } \lambda_0}{2 + z^2(1 + |x|^2)} \quad , \tag{112}$$

where $z \equiv \Delta m/\Gamma$. Thus, there is an optimum value of z for observing a given time-integrated asymmetry, depending on the value of $|x|$. For $|x| = 1$ (as happens when f is a CP eigenstate, for instance), this optimum value is $z = 1$. The case of $B^0 - \bar{B}^0$ mixing ($z \simeq 0.75$ is not far from this value.

3. Time-dependent asymmetry. The decays of neutral B mesons are expressed as a function of proper time as

$$d\Gamma(B^0_{\text{phys}} \to f)dt \sim |f_+(t) + \lambda_0 f_-(t)|^2 \quad ;$$

$$d\Gamma(\bar{B}^0_{\text{phys}} \to f)dt \sim |f_+(t) + \bar{\lambda}_0 f_-(t)|^2 \quad . \tag{113}$$

A great simplification occurs when f is a CP eigenstate: $\bar{f} = \pm f$. In this case $x^* = x^{-1}$, so $|x| = 1$, hence $|\bar{\lambda}_0| = |\lambda_0| = 1$ and $\bar{\lambda}_0 = \lambda_0^*$. We have

$$|f_+ + \lambda f_-|^2 = e^{-\Gamma t}\left|\cos\frac{\Delta m t}{2} + i\lambda_0 \sin\frac{\Delta m t}{2}\right|^2$$

$$= e^{-\Gamma t}\left[1 + 2\text{Re}(i\lambda_0)\sin\frac{\Delta m t}{2}\cos\frac{\Delta m t}{2}\right] = e^{-\Gamma t}\left[1 - \text{Im}\lambda_0 \sin(\Delta m t)\right] \tag{114}$$

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or

$$dΓ(B^0_{\text{phys}}(t) → f)/dt \sim e^{-Γt}[1 - \text{Im}λ_0 \sin(Δmt)]$$

$$dΓ(\bar{B}^0_{\text{phys}}(t) → f)/dt \sim e^{-Γt}[1 + \text{Im}λ_0 \sin(Δmt)] \quad , \tag{115}$$

where we have again neglected $ΔΓ/Γ$ in comparison with $Δm/Γ$. This step is justified for B's but not for K's. In contrast to the situation for K's, the final states to which both B and $\bar{B}$ can decay are only a small fraction of those to which B or $\bar{B}$ normally decay, and so (in accord with Eq. (23), which give rise to $ΔΓ$) one should expect quite similar lifetimes $Γ_H$ and $Γ_L$ for the two mass eigenstates.

Integration of (115) gives

$$C_f = \frac{-(Δm/Γ)\text{Im}λ_0}{1 + (Δm/Γ)^2} \tag{116}$$

for the total asymmetry (in accord with Eq. (112) for $|x| = 1$). This asymmetry is suppressed both when $Δm/Γ$ is very small (e.g., for D^0) and when it is very large (e.g., as is expected for B_s). For B_s, in order to see an asymmetry, one must not integrate with respect to time. Many experiments are planned with detection of B_s as their focus; they will require precise vertex detection to measure mixing as a function of proper time.

When more than one eigenchannel contributes to a decay, there can appear terms of the form $\cos(Δmt)$ as well as $\sin(Δmt)$ in results analogous to Eqs. (115).[127] These complicate the analysis somewhat, but information can be obtained from them[128] on the relative contributions of various channels to decays.

C. CP-violating B meson decays

Two main avenues for detecting CP violating in systems involving b quarks involve 1) decays to CP eigenstates,[17] and 2) decays to CP non-eigenstates. In both cases, partial rates for particle and antiparticle decays are compared, but experimental aspects and interpretations differ.

1. In decays to CP eigenstates, one compares the partial rate for a decay of an initial B^0 with that for an initial $\bar{B}^0$. The decays are described by time-dependent functions whose difference when integrated over all time is responsible for the rate asymmetry. We have formulated this time-dependence above, and will illustrate the calculation of the asymmetry in some specific examples.

The interference of direct decays (such as $B^0 → J/ψK_S$) and those involving mixing (such as $B^0 → \bar{B}^0 → J/ψK_S$) gives rise to rate asymmetries which can be easily interpreted in terms of the angles $α$, $β$, $γ$. Thus, if we define

$$A(f) \equiv \frac{Γ(B_{t=0} → f) - Γ(\bar{B}_{t=0} → f)}{Γ(B_{t=0} → f) + Γ(\bar{B}_{t=0} → f)} \quad , \tag{117}$$

we have, in the limit of a single direct contribution to decay amplitudes,

$$A(J/ψK_S, π^+π^-) = -\frac{x_d}{1 + x_d^2} \sin(2β, 2α) \quad , \tag{118}$$

where $x \equiv Δm/Γ$. This limit is expected to be very good for $J/ψK_S$, but some correction for penguin contributions is probably needed for $π^+π^-$. The value $x_d = 0.75 \pm 0.05$ is nearly optimum to maximize the coefficient of $\sin(2β, 2α)$.

We now give some details of the calculation of the asymmetries in Eq. (118).[23] The decay amplitudes for $B^0 → J/ψ$ are governed by the subprocesses

$$\bar{B}^0: \quad a(b → c\bar{c}s) \sim V_{cb}V^*_{cs} \quad ; \quad B^0: \quad a(\bar{b} → \bar{c}c\bar{s}) \sim V^*_{cb}V_{cs} \quad . \tag{119}$$

We are interested in the parameter $x \equiv \langle J/ψK_S|\bar{B}^0\rangle/\langle J/ψK_S|B^0\rangle$. We express K_S in terms of K and $\bar{K}$: $|K_S\rangle = p_K|K\rangle + q_K|\bar{K}\rangle$, so $\langle K_S|\bar{K}\rangle = q^*_K$; $\langle K_S|K\rangle = p^*_K$; and

$$x = \left(\frac{q}{p}\right)^*_K \frac{\langle J/ψ\bar{K}|\bar{B}\rangle}{\langle J/ψK|B\rangle} \quad . \tag{120}$$

Let us, for the moment, imagine $(q/p)_K$ to be dominated by the charmed quark loop. The value of $(q/p)_K^*$ is then $V_{cd}^* V_{cs}/V_{cd}V_{cs}^*$. The ratio of matrix elements in (10.23) is $V_{cb}V_{cs}^*/V_{cb}^* V_{cs}$. Finally, the CP-violating asymmetry depends on $\lambda_0 \equiv x(q/p)_B$, so

$$\lambda_0 \simeq \frac{V_{cd}^* V_{cs}}{V_{cd}V_{cs}^*} \frac{V_{cb}V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{td}V_{tb}^*}{V_{td}^* V_{tb}} \quad, \tag{121}$$

where we have used the dominance of the t quark loop in $B - \bar{B}$ mixing when calculating $(q/p)_B$. Cancelling some terms, we find that λ_0 is manifestly invariant under redefinition of quark phases. A similar conclusion would be drawn if ϵ_K were dominated by the top quark loop. In fact, the phase of $(q/p)_K^*$ may be calculated in either manner if one neglects m_u in comparison with m_c and m_t. In practical calculations, $(q/p)_K = (1 - \epsilon)/(1 + \epsilon)$ is very close to 1, so the phase of the first term in (121) is negligible in comparison with that of the other two.

The result is that $\lambda_0 \simeq V_{td}/V_{td}^* = e^{-2i\beta}$, where β is the angle of the unitarity triangle (Fig. 2) opposite the side corresponding to V_{ub}^*. The CP-violating asymmetry, whether time-dependent or time-integrated, measures Im $\lambda_0 = -\sin(2\beta)$, i.e., the "best-known" angle of the triangle.

Let us assume that the decay $\bar{B}^0 (\equiv b\bar{d}) \to \pi^+\pi^-$ is dominated by a process like that shown in Fig. 6, where the current creates a π^-, and the π^+ is composed of the u quark in the subprocess $b \to ud\bar{u}$ and the $\bar{d}$ quark from the initial $\bar{B}^0$. We shall neglect the effects of penguin graphs, though they could be significant.[127] (Methods have been proposed for detecting such contributions, based on searches for a $\cos(\Delta mt)$ term characteristic of the presence of more than one eigenchannel contributing to a decay to a CP eigenstate like $\pi^+\pi^-$.)

Proceeding as for the $J/\psi K_S$ case, we compare $A(\bar{B}^0 \to \pi^+\pi^-) \sim V_{ub}V_{ud}^*$ with $A(B^0 \to \pi^+\pi^-) \sim V_{ub}^* V_{ud}$, so $x = V_{ub}V_{ud}^*/V_{ub}^* V_{ud}$ and

$$\lambda_0 = \frac{q}{p}x = \frac{V_{td}V_{tb}^* V_{ub}V_{ud}^*}{V_{td}^* V_{tb}V_{ub}^* V_{ud}} \simeq \frac{V_{ub}V_{td}}{V_{ub}^* V_{td}^*} = e^{-2i(\gamma+\beta)} = e^{-2i(\pi-\alpha)} = e^{2i\alpha} \quad. \tag{122}$$

Then Im$\lambda_0 = \sin(2\alpha)$, which can have either sign, depending on the shape of the unitarity triangle.

For the range of parameters noted in Fig. 5, we expect $-0.4 \leq A(J/\psi K_S) \leq -0.1$, i.e., an asymmetry of a definite sign, and $-0.48 \leq A(\pi^+\pi^-) \leq 0.3$, i.e., nearly any asymmetry within the possible limits imposed by the factor $x_d/(1 + x_d^2)$. Thus, the $J/\psi K_S$ asymmetry is likely to provide a consistency check, while the $\pi^+\pi^-$ asymmetry should be more useful in specifying the parameters ρ, η (unless it lies outside the expected limits).

In order to employ this method it is necessary to know the flavor of the produced B meson. We shall remark presently on possible "tagging" methods.

2. Decays to CP non-eigenstates can exhibit rate asymmetries only if there are two different weak decay amplitudes *and* two different strong phase shifts associated with them. Comparing

$$\Gamma \equiv \Gamma(B^0 \to f) = |a_1 e^{i(\phi_1+\delta_1)} + a_2 e^{i(\phi_2+\delta_2)}|^2 \tag{123}$$

with

$$\bar{\Gamma} \equiv \Gamma(\bar{B}^0 \to \bar{f}) = |a_1 e^{i(-\phi_1+\delta_1)} + a_2 e^{i(-\phi_2+\delta_2)}|^2 \tag{124}$$

we notice that only the weak phases ϕ_i change sign under CP inversion, not the strong phases δ_i. Defining $\phi \equiv \phi_1 - \phi_2$, $\delta \equiv \delta_1 - \delta_2$, we find

$$\Gamma - \bar{\Gamma} \propto 2a_1 a_2 \sin\phi \sin\delta \quad, \tag{125}$$

so both ϕ and δ must be not equal to 0 or π in order to see a rate difference.

A pair of channels in which both weak and strong phase shift differences might be nonzero, for instance, could be the $I = 1/2$ and $I = 3/2$ amplitudes for $B \to \pi K$. Here one is able to compare decays of charged B mesons with those of their antiparticles, so the identification of the flavor of the

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decaying meson does not pose a problem. On the other hand, δ is generally expected to be small and quite uncertain for the energies characteristic of B decays. We shall outline below some recent progress in using decays of charged B mesons to provide information on CKM phases *without* necessarily having to observe a CP-violating decay rate asymmetry.

D. B Flavor tagging in $C = \pm 1$ $B\bar{B}$ states

In the decays of neutral B mesons to CP eigenstates, it is necessary to know the flavor of the meson at time of production. A conventional means for "tagging" the flavor of a B is to identify the flavor of the meson produced in association with it. At a hadron collider or in high energy e^+e^- collisions as at LEP, this method suffers only from the possible dilution of the "tagging" signal by $B^0 - \bar{B}^0$ mixing, and from the difficulty of finding the "tagging" hadron.

In e^+e^- annihilations, one frequently can produce a $B\bar{B}$ state with a definite eigenvalue of the charge-conjugation operator C. For example, at the $\Upsilon(4S)$, $C(B\bar{B}) = -1$, since the pair is produced from a virtual photon, while the production of a $B^*\bar{B}$ pair by a virtual photon followed by $B^* \to \gamma + B$ leads to a $B\bar{B}$ pair with $C = +1$.

We present an abbreviated discussion, of which a more complete version can be found elsewhere.[129] Consider the process in which an e^+e^- collision produces a $B^0\bar{B}^0$ pair, one of which decays to $J/\psi K_S$ and the other of which decays semileptonically. We can tell what the flavor is of the meson decaying semileptonically, since $\bar{b}$ (in a B^0) gives $\bar{c}$ (in D^{*-})$\ell^+\bar{\nu}_\ell$, while b (in a $\bar{B}^0$) gives c (in D^{*+})$\ell^-\nu_\ell$. However, because of the possibility of $B^0 \leftrightarrow \bar{B}^0$ mixing, the flavor of the meson decaying to $J/\psi K_S$ is not uniquely labelled.

We can write the time-dependent partial decay rates as

$$d^2\Gamma[J/\psi K_S(t) \ D^{*+}\ell^-\bar{\nu}_\ell(\bar{t})]_{C=\mp 1}/dtd\bar{t} \sim e^{-\Gamma(t+\bar{t})}[1 \mp \sin \Delta m(t \mp \bar{t})\mathrm{Im}\lambda] \quad ;$$

$$d^2\Gamma[J/\psi K_S(t) \ D^{*-}\ell^+\nu_\ell(\bar{t})]_{C=\mp 1}/dtd\bar{t} \sim e^{-\Gamma(t+\bar{t})}[1 \pm \sin \Delta m(t \mp \bar{t})\mathrm{Im}\lambda] \quad . \tag{126}$$

The term contributing to the time-dependent asymmetry is even or odd in $t - \bar{t}$ depending on whether $C(B\bar{B})$ is even or odd. Thus, for a $C = -$ eigenstate [such as $\Upsilon(4S)$], the time-integrated asymmetry vanishes. This curtails the utility of the $\Upsilon(4S)$ in producing $B\bar{B}$ pairs for the study of CP-violating asymmetries, unless time-dependent effects can be studied. If we can distinguish which of the times t and $\bar{t}$ is earlier, we can study an asymmetry proportional to

$$\Gamma^2 \int_0^\infty dtd\bar{t}e^{-\Gamma(t+\bar{t})} \sin \Delta m(|t - \bar{t}|) = \frac{z}{1 + z^2} \simeq 0.48 \ \text{ for } \ z \equiv \Delta m/\Gamma \simeq 0.75 \ . \tag{127}$$

For a $C = +$ eigenstate [as might be reachable just above the $\Upsilon(4S)$ via $e^+e^- \to B\bar{B}^* \to B\bar{B}\gamma$], the time-integrated asymmetry is proportional to

$$\Gamma^2 \int_0^\infty dtd\bar{t}e^{-\Gamma(t+\bar{t})} \sin \Delta m(|t + \bar{t}|) = \frac{2z}{(1 + z^2)^2} \simeq 0.61 \ \text{ for } \ z \equiv \Delta m/\Gamma \simeq 0.75 \ . \tag{128}$$

The need to distinguish which of the times t and $\bar{t}$ is earlier has been addressed by the construction of asymmetric "B factories" at SLAC (Stanford) and KEK (Tsukuba), in which two beams of unequal energies collide, leading to a Lorentz boost of the center of mass. It may also be possible to employ ingenious schemes[130] for enhancing vertex resolution in symmetric machines such as CESR (Cornell).

E.   Flavor tagging by means of correlations with "nearby" hadrons

In this section I would like to discuss recent progress in tagging neutral B mesons by means of the hadrons produced nearby in phase space. This method, also proposed[131] for tagging strange B's

via associated kaons, has been the subject of recent papers devoted to correlations of nonstrange B's with charged pions.[132]

1. Fragmentation vs. resonances. The existence of correlations between B mesons and pions can be visualized either in terms of a fragmentation picture or in terms of explicit resonances.

In a fragmentation picture, if a b quark picks up a $\bar{d}$ quark from the vacuum to become a $\bar{B}^0$ meson, and a charged pion containing the corresponding d quark is generated, that pion will be a π^-. It is likely to lie "near" the $\bar{B}^0$ in phase space, in the sense that its transverse momentum with respect to the B is low, its rapidity is correlated with that of the B, or the effective mass of the πB system is low. Similarly, if a $\bar{b}$ quark picks up a d quark to become a B^0, the charged pion containing the corresponding $\bar{d}$ will be a π^+.

If a b quark picks up a strange antiquark to become a $\bar{B}_s$, the next hadron down the fragmentation chain will contain an s quark. If this hadron is a charged kaon, it will be a K^-.[131] A $B_s = \bar{b}s$ will correspondingly be associated with a K^+. By similar arguments[133] a $\bar{B}^0 = b\bar{d}$ can be associated with a proton $p = uud$, while a $B^0 = \bar{b}d$ can be associated with an antiproton $\bar{p} = \bar{u}\bar{u}\bar{d}$.

The signs of the pions in the above correlations are those which would have resulted from the decays $B^{**+} \to B^{(*)0}\pi^+$ or $B^{**-} \to \bar{B}^{(*)0}\pi^-$. We utilize the double-asterisk superscript to distinguish B^{**}'s from the hyperfine partners of the B's, the B^*'s, which are only about 46 MeV heavier than the B's and cannot decay to them via pions.

The importance of explicit *narrow* B^{**} resonances is that they permit reduction of combinatorial backgrounds. Thus, we turn to what is expected (and, more recently, observed) about such resonances.

2. Spectroscopic predictions. We shall briefly recapitulate material which has been presented in more detail elsewhere.[26,28] In a hadron containing a single heavy quark, that quark ($Q = c$ or b) plays the role of an atomic nucleus, with the light degrees of freedom (quarks, antiquarks, gluons) analogous to the electron cloud. The properties of hadrons containing b quarks then can calculated from the corresponding properties of charmed particles by taking account[68] of a few simple "isotope effects." If q denotes a light antiquark, the mass of a $Q\bar{q}$ meson can be expressed as

$$M(Q\bar{q}) = m_Q + \text{const.}[n, L] + \frac{\langle p^2 \rangle}{2m_Q} + a\frac{\langle \sigma_{\mathbf{q}} \cdot \sigma_{\mathbf{Q}} \rangle}{m_q m_Q} + \mathcal{O}(m_Q^{-2}) \quad . \tag{129}$$

Here the constant depends only on the radial and orbital quantum numbers n and L. The $\langle p^2 \rangle/2m_Q$ term expresses the dependence of the heavy quark's kinetic energy on m_Q, while the last term is a hyperfine interaction. The expectation value of $\langle \sigma_{\mathbf{q}} \cdot \sigma_{\mathbf{Q}} \rangle$ is $(+1, -3)$ for $J^P = (1^-, 0^-)$ mesons. If we define $\bar{M} \equiv [3M(1^-) + M(0^-)]/4$, we find

$$m_b - m_c + \frac{\langle p^2 \rangle}{2m_b} - \frac{\langle p^2 \rangle}{2m_c} = \bar{M}(B\bar{q}) - \bar{M}(c\bar{q}) \simeq 3.34 \text{ GeV} \quad . \tag{130}$$

so $m_b - m_c > 3.34$ GeV, since $\langle p^2 \rangle > 0$. Details of interest include (1) the effects of replacing nonstrange quarks with strange ones, (2) the energies associated with orbital excitations, (3) the size of the $\langle p^2 \rangle$ term, and (4) the magnitude of hyperfine effects. In all cases there exist ways of using information about charmed hadrons to predict the properties of the corresponding B hadrons. A recent comparison of the charmed and beauty spectra may be found in Ref..[28] For S-wave states the predictions of the heavy-quark symmetry approach work rather well.

The $B^* - B$ hyperfine splitting scales as the inverse of the heavy-quark mass: $B^* - B = (m_c/m_b)(D^* - D)$. Consequently, while $D^{*+} \to D^0\pi^+$ and $D^{*+} \to D^+\pi^0$ are both allowed, leading to a useful method[134] for identifying charmed mesons via the soft pions often accompanying them, the only allowed decay of a B^* is to $B\gamma$. No soft pions are expected to accompany B mesons. One must look to the next-higher set of levels, the B^{**} resonances consisting of a $\bar{b}$ quark and a light quark in a P-wave, or the fragmentation process mentioned above, for the source of pions correlated with the flavor of B mesons.

Table 4. P-wave resonances of a heavy antiquark and a light quark $q = u$, d. In final states P, V denote a heavy 0^-, 1^- meson. For strange states, add about 0.1 GeV/c^2 to the masses.

J_j^P	$\bar{c}q$ mass (GeV/c^2)	$\bar{b}q$ Mass (GeV/c^2)	Allowed final state(s)
$2_{3/2}^+$	$2.46^{a)}$	$\sim 5.77^{b)}$	$P\pi, V\pi$
$1_{3/2}^+$	$2.42^{a)}$	$\sim 5.77^{b)}$	$V\pi$
$1_{1/2}^+$	$< 2.42^{c)}$	$< 5.77^{b)}$	$V\pi$
$0_{1/2}^+$	$< 2.42^{c)}$	$< 5.77^{b)}$	$P\pi$

$^{a)}$ Observed value..[136]
$^{b)}$ Predicted by extrapolation from corresponding D^{**} using heavy-quark symmetry.
$^{c)}$ Predicted in most quark models.

One can use heavy-quark symmetry or explicit quark models to extrapolate from the properties of known D^{**} resonances to those of B^{**} states. Two classes of such resonances are expected,[135] depending on whether the total angular momentum $j = s_q + L$ of the light quark system is 1/2 or 3/2. Here s_q is the light quark's spin and $L = 1$ is its orbital angular momentum with respect to the heavy antiquark. The light quark's $j = 1/2$, 3/2 can couple with the heavy antiquark's spin $S_{\bar{Q}} = 1/2$ to form states with total angular momentum and parity $J_j^P = 0_{1/2}^+, 1_{1/2}^+, 1_{3/2}^+, 2_{3/2}^+$.

The $0_{1/2}^+$ and $1_{1/2}^+$ states are expected to decay to a ground-state heavy meson with $J^P = 0^-$ or 1^- and a pion via an S-wave, and hence to be quite broad. No evidence for these states exists in the $\bar{c}q$ or the $\bar{b}q$ system. By contrast, the $1_{3/2}^+$ and $2_{3/2}^+$ states are expected to decay to a ground-state heavy meson and a pion mainly via a D-wave, and hence to be narrow. Candidates for all the nonstrange and strange D^{**} states of this variety have been identified.[136] The known nonstrange D^{**} resonances (identified in both charged and neutral states) are a 2^+ state around 2.46 GeV/c^2, decaying to $D\pi$ and $D^*\pi$, and a 1^+ state around 2.42 GeV/c^2, decaying to $D^*\pi$. In addition, strange D_s^{**+} resonances have been seen, at 2.535 GeV/c^2 (a candidate for $1_{3/2}^+$) and 2.573 GeV/c^2 (a candidate for $2_{3/2}^+$). Thus, adding a strange quark adds about 0.1 GeV/c^2 to the mass.

Once the masses of D^{**} resonances are known, one can estimate those of the corresponding B^{**} states by adding about 3.32 GeV (the quark mass difference minus a small binding correction). The results of this exercise are shown in Table 4. The reader should consult Ref.[137] for more detailed predictions based on potential models and for relations between decay widths of D^{**} states and those of the B^{**}'s. Thus, the study of excited charmed states can play a crucial role in determining the feasibility of methods for identifying the flavor of neutral B mesons.

3. Spectroscopic observations

The OPAL,[138] DELPHI,[139] and ALEPH[140] Collaborations have now observed $B\pi$ correlations which can be interpreted in terms of the predicted $J_j^P = 1_{3/2}^+, 2_{3/2}^+$ states. The OPAL data are shown in Fig. 8, while the DELPHI data are shown in Fig. 9. (The ALEPH data were presented since the Summer School.) Note that OPAL also sees a BK correlation.

In all experiments one is able to measure only the effective mass of a $B\pi$ system. If a B^{**} decays to $B^*\pi$, the photon in the $B^* \to \gamma B$ decay is lost, leading to an underestimate of the B^{**} mass by about 46 MeV/c^2 but negligible energy smearing.[132] Thus, the contributions to the $B\pi$ mass distribution of 1^+ and 2^+ resonances with spacing $\delta \equiv M(2^+) - M(1^+)$ can appear as three peaks, one at $M(2^+)$ due to $2^+ \to B\pi$, one at $M(2^+) - 46$ MeV/c^2 due to $2^+ \to B^*\pi$, and one at $M(2^+) - \delta - 46$ MeV/c^2 due to $1^+ \to B^*\pi$.

The OPAL Collaboration fits their $B\pi$ mass distribution either with one peak with $M =$
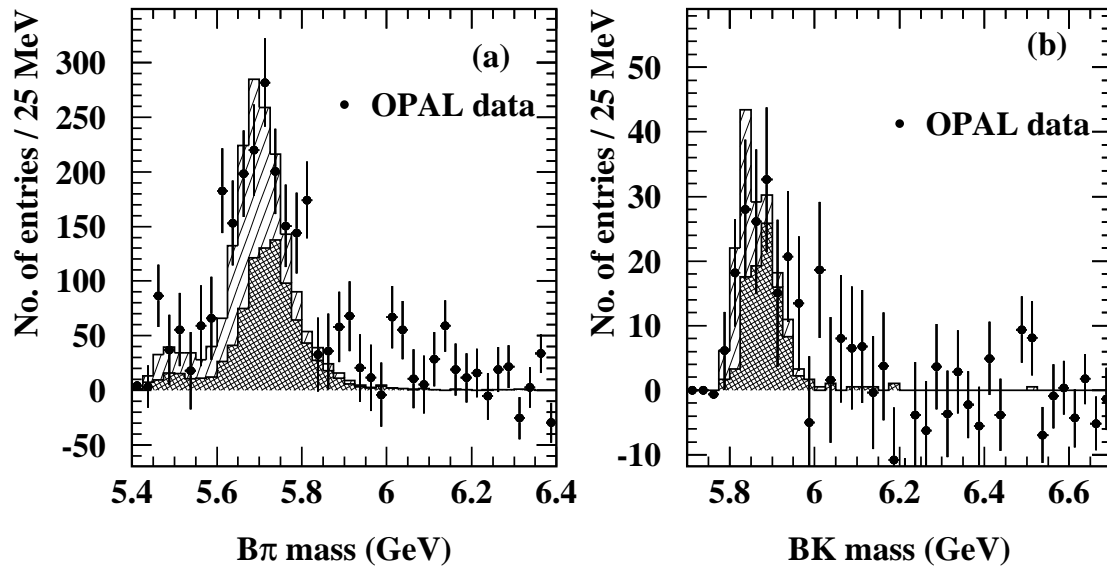
Fig. 8. Evidence for B** resonances obtained by the OPAL Collaboration.[138] (a) $B\pi$ background-subtracted mass distribution; (b) BK background-subtracted mass distribution. The solid and cross-hatched histograms correspond to the contributions of 2^+ and 1^+ resonances in two-resonance fits based on the mass splittings and branching ratios predicted in Ref..[137]

5681 ± 11 MeV/c^2 and width $\Gamma = 116 \pm 24$ MeV, or two resonances, a 1^+ candidiate at 5725 MeV/c^2 with width $\Gamma = 20$ MeV and a 2^+ candidiate at 5737 MeV/c^2 with width $\Gamma = 25$ MeV. The widths, mass splittings, and branching ratios to $B\pi$ and $B^*\pi$ in this last fit are taken from Ref.,[137] and only the overall mass and production cross sections are left as free parameters. The OPAL BK mass distribution is fit either with a single resonance at $M = 5853 \pm 15$ MeV/c^2 with width $\Gamma = 47 \pm 22$ MeV, or two narrow resonances, a 1^+ candidate at 5874 MeV/c^2 and a 2^+ candidate at 5886 MeV/c^2.

The fitted masses of the nonstrange and strange resonances are respectively 30 MeV/c^2 lower and 40 MeV/c^2 higher than the predictions of Ref..[137] The difference could well be due to additional contributions to the nonstrange channel from the lower-lying $0^+_{1/2}$ and $1^+_{1/2}$ states or from nonresonant fragmentation. The corresponding strange states might lie below BK threshold.

The OPAL results imply that $(18 \pm 4)\%$ of the observed B^+ mesons are accompanied by a "tagging π^-" arising from B^{**0} decay. By isospin reflection, one should then expect $(18 \pm 4)\%$ of B^0 to be accompanied by a "tagging π^+" arising from B^{**+} decay. This is good news for the possibility of "same-side tagging" of neutral B mesons. [Another $(2.6 \pm 0.8)\%$ of the observed B^+ mesons are accompanied by a "tagging K^-." The isospin-reflected kaon is neutral, and unsuitable for tagging.]

The DELPHI data can be fit with a single peak having a mass of $M = 5732 \pm 5 \pm 20$ MeV/c^2 and width $\Gamma = 145 \pm 28$ MeV. The number of $B^{**}_{u,d}$ per b jet is quoted as $0.27 \pm 0.02 \pm 0.06$. The ALEPH peak occurs at $M(B\pi) - M(B) = 424 \pm 4 \pm 10$ MeV/c^2, with Gaussian width $\sigma = 53 \pm 3 \pm 9$ MeV/c^2. The ALEPH B^{**} signal is characterized by a production rate

$$\frac{B(Z \to b \to B^{**}_{u,d})}{B(Z \to b \to B_{u,d})} = (27.9 \pm 1.6 \pm 5.9 \pm 3.8)\% \quad , \tag{131}$$

where the first error is statistical, the second is systematic, and the third is associated with uncertainty in ascribing the peak to the contribution of various resonances. Multiplying the DELPHI and ALEPH results by the isospin factor of 2/3 to compare with the OPAL result, we find complete agreement among the three.
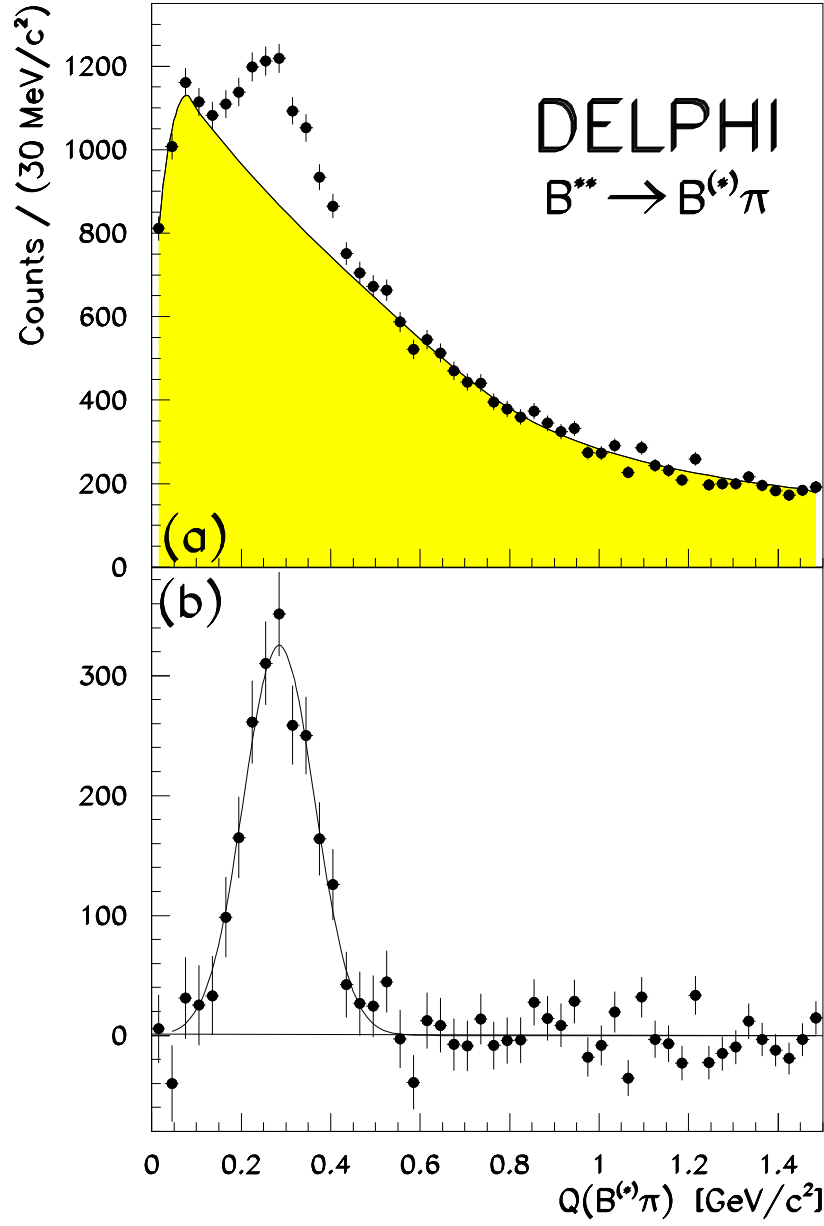
Fig. 9. Evidence for B** resonances obtained by the DELPHI Collaboration.[139] (a) Unsubtracted distribution in $Q(B^{(*)}\pi) \equiv M(B\pi) - M_B - m_\pi$. (b) Background-subtracted distribution.

4. Isospin and Correlations

In principle it should be possible to calibrate the correlations between charged pions and neutral B's by comparing them with the isospin-reflected correlations betwen charged pions and *charged B*'s (whose flavor may be easier to identify). Thus, the enhancement of the non-exotic $\pi^+ B^0$ channel with respect to the exotic $\pi^- B^0$ channel should be the same as that of the non-exotic $\pi^- B^+$ channel with respect to the exotic $\pi^+ B^+$ channel. What can spoil this relation? I. Dunietz and I[141] have explored several instances in which care is warranted in making this comparison. Some of the differences could be real, but there are many sources of potential instrumental error against which one has concrete remedies.

a) Interaction with the producing system can lead to final states which need not be invariant under isospin reflection. For example, although a pair of gluons would produce a $b\bar{b}$ pair with isospin $I = 0$, the fragmentation process could involve picking up quarks from the producing system (e.g., proton or antiproton fragments) in a manner not invariant with respect to $u \leftrightarrow d$ substitution. Similarly, the production of a B meson through diffractive dissociation of a proton (which has more valence u quarks than d quarks) need not be invariant under isospin reflection.

b) Misidentification of associated charged kaons as pions can lead one to overestimate the charged B – charged pion correlations. One expects $B^+ K^-$ correlations, as seen by OPAL, but not $B^0 K^+$ correlations. As mentioned, the isospin reflection of a charged kaon is neutral, and would not contribute to a correlation between charged particles and neutral B's.

c) Pions in the decay of the associated B will not be produced in an isospin-reflection-symmetric manner. One must be careful not to confuse them with primary pions.

d) Different time-dependent selection criteria for charged and neutral B's can lead one to mis-estimate the mixing of neutral B's with their antiparticles. (I thank P. Derwent for pointing this out.) It is possible to make an unfortunate cut on B^0 lifetime which enhances the mixing considerably with respect to the value obtained by integrating over all times.

e) Overestimates of particle identification efficiencies can lead to confusion in identification of the flavor of a neutral B through the decay $B^0 \to J/\psi K^{*0} \to J/\psi K^+ \pi^-$. It is possible, especially for K^+ and π^- with equal laboratory momenta, to confuse them with π^+ and K^-, while still keeping them in a K^* peak.

The CDF Collaboration at Fermilab has been studying charged pion – B correlations ever since the reports of Ref.[132] appeared, but no public announcement of these results has yet appeared. Such correlations certainly should be present in hadron collider data.

F. B decays to pair of light mesons

In this subsection we turn to decays of B mesons to CP non-eigenstates. We have suggested[142] that relations between decays of charged B's to pairs of light pseudoscalar mesons based on flavor SU(3)[143] could provide information on weak phases by means of *rate measurements alone*. The latest chapter in this story has been written since the Summer School.

1. $\pi\pi$ and πK final states. Two years ago the CLEO Collaboration[144] presented evidence for a combination of $B^0 \to K^+\pi^-$ and $\pi^+\pi^-$ decays, generically known as $B^0 \to h^+\pi^-$. On the basis of 2.4 fb^{-1} of data, the most recent result[145] is $B(B^0 \to h^+\pi^-) = (1.81\ ^{+0.6}_{-0.5}\ ^{+0.2}_{-0.3} \pm 0.2) \times 10^{-5}$. Although one still cannot conclude that either decay mode is nonzero at the 3σ level, the most likely solution is roughly equal branching ratios (i.e., about 10^{-5}) for each mode.

Other results[146] of the CLEO Collaboration on related modes include the upper bounds $B(B^0 \to \pi^0\pi^0) < 1.0 \times 10^{-5}$, $B(B^+ \to \pi^+\pi^0) < 2.3 \times 10^{-5}$, and $B(B^+ \to K^+\pi^0) < 3.2 \times 10^{-5}$. Interesting levels for the last two modes[142] are probably around $(1/2) \times 10^{-5}$, and probably 10^{-6} or less for $\pi^0\pi^0$. With good particle identification and a factor of several times more data, it appears that CLEO will be able to make a systematic study of decay modes with two light pseudoscalars. What can it teach us?

2. SU(3) relations and the phase γ. We mentioned earlier that rate asymmetries in the decays $B^0 \to \pi^+\pi^-$ and $\bar{B}^0 \to \pi^+\pi^-$ could provide information on the weak angle α, as long as a single
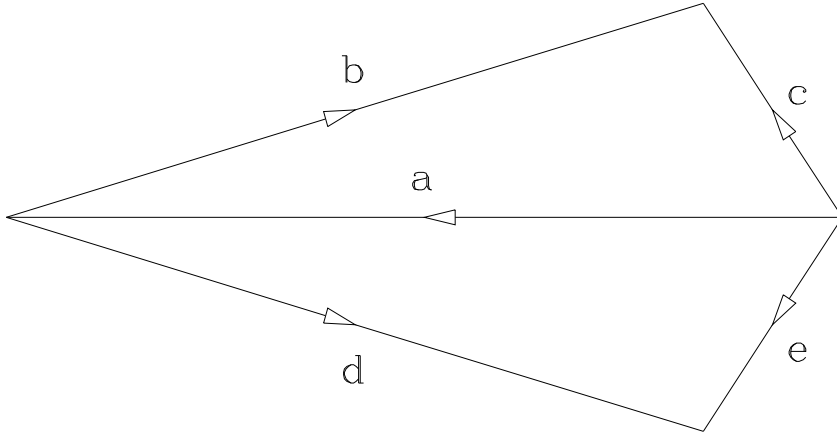
Fig. 10. Amplitude triangles illustrating the extraction of the weak phase γ from charged B decays to $\pi\pi$ and πK. (a) $A(B^+ \to \pi^+ K^0) = A(B^- \to \pi^- \bar{K}^0)$; (b) $\sqrt{2}A(B^+ \to \pi^0 K^+)$; (c) $\sqrt{2}\tilde{r}_u A(B^+ \to \pi^+\pi^0)$; (d) $\sqrt{2}A(B^- \to \pi^0 K^-)$; (e) $\sqrt{2}\tilde{r}_u A(B^- \to \pi^-\pi^0)$. The exterior angle between sides c and e is 2γ.

quark subprocess dominated the decay. Additional contributions from penguin diagrams[127] can be taken into account by means of an isospin triangle construction[128] involving the relation $A(B^0 \to \pi^+\pi^-) = \sqrt{2}A(B^0 \to \pi^0\pi^0) + \sqrt{2}A(B^+ \to \pi^+\pi^0)$, and the corresponding relation for the charge-conjugate processes. Here we define amplitudes such that a partial width is always proportional to the square of an amplitude with the same coefficient.

An amplitude quadrangle applies to the decays $B \to \pi K$: $A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = A(B^0 \to \pi^- K^+) + \sqrt{2}A(B^0 \to \pi^0 K^0)$. When combined with the corresponding relation for $\bar{B}^0$ and B^- decays, and used in conjunction with the time-dependence of the decays $(B^0$ or $\bar{B}^0) \to \pi^0 K_S$, these quadrangles are useful in extracting the weak phase α.[147]

In examining SU(3) relations among $B \to PP$ amplitudes, where P is a pseudoscalar meson, we found that one of the diagonals of the amplitude quadrangle for $B \to \pi K$ (corresponding to an amplitude with isospin $I = 3/2$) could be related to the purely $I = 2$ amplitude for $B^+ \to \pi^+\pi^0$. We obtained the relation

$$A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = \tilde{r}_u\sqrt{2}A(B^+ \to \pi^+\pi^0) \quad , \tag{132}$$

where $\tilde{r}_u \equiv (f_K/f_\pi)|V_{us}/V_{ud}|$. The $B^+ \to \pi^+ K^0$ amplitude is expected to be dominated by a (gluonic) penguin contribution, involving mainly the combination of CKM elements $V_{tb}^* V_{ts}$, whose electroweak phase is π. The electroweak phase of the $B^+ \to \pi^+\pi^0$ amplitude is just $\gamma = \text{Arg}(V_{ub}^*)$. Thus, in the absence of strong-interaction phase shift differences, the shape of the amplitude triangle would give γ. One could account for strong-interaction phases by comparing the amplitude triangle for B^+ decays with that for B^- decays.[142] This construction is illustrated in Fig. 10. The angle between the sides corresponding to $B^+ \to \pi^+\pi^0$ and $B^- \to \pi^-\pi^0$ is 2γ. The most likely of two possible orientations for the triangles is shown; the other orientation corresponds to reflection of the B^- triangle about the common base.

The amplitude $A(\pi^0 K^+)$ which is the sum of "tree" and penguin graph contributions can be expressed[148] as

$$\sqrt{2}A(\pi^0 K^+) = Ae^{i\gamma}e^{i\delta_3} + Be^{i\delta_P} = e^{i\delta_P}(Ae^{i\gamma}e^{i\delta} + B) \quad , \tag{133}$$

where δ_3 is the strong $I = 3/2$ phase, δ_P is the strong penguin phase, and $\delta \equiv \delta_3 - \delta_P$. In the corresponding expression for $A(\pi^0 K^-)$, only the sign of γ is changed. The $\pi^0 K^\pm$ decay rates are then

$$\Gamma(\pi^0 K^\pm) = (1/2)[A^2 + B^2 + 2AB\cos(\delta \pm \gamma)] \quad . \tag{134}$$

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The sum and difference of these rates are

$$S \equiv \Gamma(\pi^0 K^+) + \Gamma(\pi^0 K^-) = A^2 + B^2 + 2AB \cos \delta \cos \gamma \quad , \tag{135}$$

$$D \equiv \Gamma(\pi^0 K^+) - \Gamma(\pi^0 K^-) = 2AB \sin \delta \sin \gamma \quad . \tag{136}$$

The CP-violating rate difference D is probably very small as a result of the likely smallness of the phase difference δ.

For $|A/B| = 1/3$, $\delta = 0$, and $\cos \gamma \simeq 0$ (realistic values), and expressing $\cos \gamma = (S - A^2 - B^2)/2AB$, we found that in order to measure γ to $10°$ one needs a sample consisting of about 100 events in the channel $\pi^0 K^\pm$ corresponding to S.[25]

3. Electroweak penguins. The analyses of Ref.[142] assumed that the only penguin contributions to B decays were gluonic in nature. Consequently, one could treat the flavor-dependence in terms of an effective $\bar{b} \to \bar{d}$ or $\bar{b} \to \bar{s}$ transition since the gluon couples to light quarks in a flavor-symmetric manner. Thus, the $I = 3/2$ amplitude in $B \to \pi K$ (the diagonal of the πK amplitude quadrangle mentioned above) was due entirely to the Cabibbo-suppressed "tree-diagram" process $\bar{b} \to \bar{u} u \bar{s}$, whose weak phase was well-specified.

It was pointed out[149,150] that in certain penguin-dominated B decays such as $B \to \pi K^*$ and πK, electroweak penguin amplitudes were large enough to compete favorably with the tree amplitude in the $I = 3/2$ channel. In contrast to gluonic penguins, the virtual photon or Z emitted in an electroweak penguin diagram does not couple to light quarks in a flavor-symmetric manner, and possesses an $I = 1$ component. Specifically, if one decomposes amplitudes into isospin channels,

$$A(B^+ \to \pi^+ K^0) = (1/3)^{1/2} A_{3/2} - (2/3)^{1/2} A_{1/2} \quad ,$$

$$A(B^+ \to \pi^0 K^+) = (2/3)^{1/2} A_{3/2} + (1/3)^{1/2} A_{1/2} \quad , \tag{137}$$

Deshpande and He[150] find, in a specific calculation, that

$$A_{1/2} \sim -0.75 e^{i\gamma} e^{i\delta_{T,1/2}} + 7.3 e^{i\delta_{P,1/2}} \quad ,$$

$$A_{3/2} \sim -1.06 e^{i\gamma} e^{i\delta_{T,3/2}} + 0.84 e^{i\delta_{P,3/2}} \quad , \tag{138}$$

where the first term in each equation is the "tree" contribution (of lowest order in electroweak interactions), while the second term is the penguin contribution. Only the electroweak penguin contributes to the $I = 3/2$ amplitude, but with magnitude comparable to the tree contribution. The electroweak penguin spoils the relation of Eq. (132).

Recently several of us re-examined the effects of SU(3) breaking[151] and electroweak penguins,[152] to see if one could extract electroweak penguin effects *directly from the data*. Since our previous SU(3) decomposition gave a complete set of reduced amplitudes, electroweak penguins only changed the interpretation of these amplitudes, so that a separation of electroweak penguin effects was not possible merely on the basis of SU(3).

As pointed out by Deshpande and He,[153] certain amplitudes [notably those for $B_s \to (\pi^0$ or $\rho^0) + (\eta$ or $\phi)$] are expected to be dominated by electroweak penguins. We noted that the πK amplitude quadrangle could be written in such a manner that one if its diagonals was equal to $\sqrt{3} A(B_s \to \pi^0 \eta_8)$, where η_8 denotes an unmixed octet member. The shape of the quadrangle, shown in Fig. 11, is uniquely determined, up to possible discrete ambiguities. The case of octet-singlet mixtures in the η requires us to replace the $\sqrt{3}$ by the appropriate coefficient; one can show that the SU(3) singlet contribution of the η is unimportant in this case.

The quadrangle has been written in such a way as to illustrate the fact[142] that the $B^+ \to \pi^+ K^0$ amplitude receives only penguin contributions in the absence of $\mathcal{O}(f_B/m_B)$ corrections. The weak phases of $\bar{b} \to \bar{s}$ penguins, which are dominated by a top quark in the loop, are expected to be π. We have oriented the quadrangle to subtract out the corresponding strong phase, and define corresponding strong phase shift differences $\tilde{\delta}$ with respect to the strong phase of the $B^+ \to \pi^+ K^0$ amplitude.
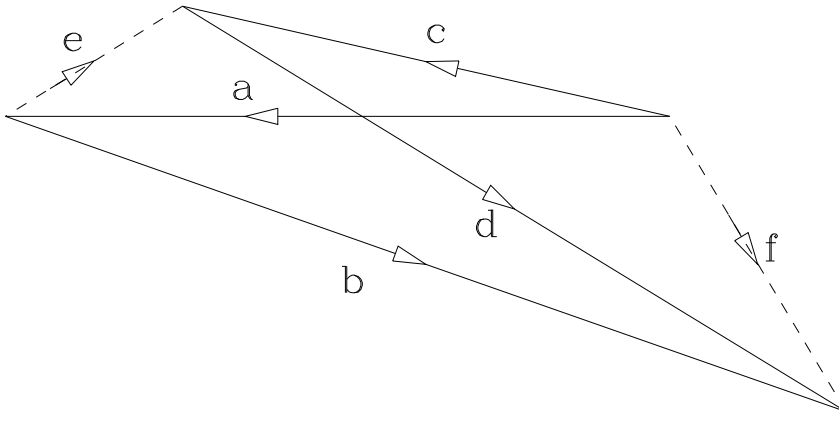
Fig. 11. Amplitude quadrangle for $B \to \pi K$ decays. (a) $A(B^+ \to \pi^+ K^0)$; (b) $\sqrt{2}A(B^+ \to \pi^0 K^+)$; (c) $\sqrt{2}A(B^0 \to \pi^0 K^0)$; (d) $A(B^0 \to \pi^- K^+)$; (e) the diagonal $D_2 = \sqrt{3}A(B_s \to \pi^0 \eta_8)$; (f) the diagonal $D_1 = A_{3/2}$ corresponding to the $I = 3/2$ amplitude.

The $I = 3/2 \, \pi K$ amplitude is composed of two parts, as noted above. We can rewrite it slightly as

$$A_{3/2} = |A_T| e^{i\gamma} e^{i\tilde{\delta}_{T,3/2}} - |A_P| \quad . \tag{139}$$

The corresponding charge-conjugate quadrangle has one diagonal equal to

$$\bar{A}_{3/2} = |A_T| e^{-i\gamma} e^{i\tilde{\delta}_{T,3/2}} - |A_P| \quad , \tag{140}$$

so that one can take the difference to eliminate the electroweak penguin contribution:

$$A_{3/2} - \bar{A}_{3/2} = 2i|A_T| \sin\gamma \; e^{i\tilde{\delta}_{T,3/2}} \quad . \tag{141}$$

The quantity $|A_T|$ can be related to the $I = 2 \, \pi\pi$ amplitude in order to obtain $\sin\gamma$. Specifically, if we neglect electroweak penguin effects in $B^+ \to \pi^+ \pi^0$ (a good approximation), we find that

$$|A_T| = \sqrt{2}\tilde{r}_u |A(B^+ \to \pi^+ \pi^0)| \quad . \tag{142}$$

Thus, we can extract not only $\sin\gamma$, but also a strong phase shift difference $\tilde{\delta}_{T,3/2}$, by comparing Eqs. (141) and (142). If such a strong phase shift difference exists, the B and $\bar{B}$ quadrangles will have different shapes, and CP violation in the B system will already have been demonstrated.

The challenge in utilizing the amplitude quadrangle in Fig. 11 is to measure $B(B_s \to \pi^0 \eta)$, which has been estimated[153] to be only 2×10^{-7}! Very recently (since the Summer School) Deshpande and He[154] have pointed out that the amplitude triangle

$$2A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = \sqrt{6}A(B^+ \to \eta_8 K^+) \quad , \tag{143}$$

implied by the SU(3) relations of Refs.,[142,152] where η_8 is the octet component of the η, permits one to specify the quantity $A_{3/2} - \bar{A}_{3/2}$ in Eq. (141) and extract γ as above. Here there is some delicacy associated with the SU(3) singlet component of the physical η.

4. Other final states.

a) PV final states ($V = \rho, \; \omega, \; K^*, \; \phi$) are characterized by more graphs (and hence more reduced SU(3) amplitudes), since one no longer has the benefit of Bose statistics as in $B \to PP$ decays. There still exist quadrangle relations in ρK and $K^* \pi$ decays, however. Remarkably, if $\Delta S = 0$ gluonic penguin diagrams (small in any case) are approximately equal for the cases in which the

Table 5. Progress in the study of CP violation in B meson systems.

Step	1/87	2/95
1. See B's	$B \to J/\psi + \ldots$ a good tag	Good signals of $B \to J/\psi$ $+K$ in hadron colliders
2. Measure branching ratios	Expected $D\pi$, $J/\psi\ K,\ldots$ level of 10^{-3}	$B(D\pi) \sim 0.3\%$ $B(J/\psi\ K) \simeq 0.1\%$
3. $B_u : B_d : B_s$ needed	Expected $\sim 2:2:1$	Still not known. CP viol. hard to see for B_s.
4. See $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing	Some UA1 evidence for $B_s - \bar{B}_s$ mixing	$(\Delta m/\Gamma)_B = 0.75 \pm 0.15$ $(\times \stackrel{>}{\sim} 15$ for $B_s)$
5. See $b \to u$ and t quark	Neither seen yet	$|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ $m_t = 180 \pm 12$ GeV$/c^2$
6. See CP violation in B system	Optimistic estimate: need $\stackrel{>}{\sim} 10^7$ B's	Realism: need $\stackrel{>}{\sim}$ (few $\times 10^8$) B's in any practical expt.
7. Measure time-dep. CP violation	Importance stressed of cuts on decay times	Asymmetric B factories under construction

spectator quark ends up in a vector meson and in a pseudoscalar [see Ref.[152] for details], the previous quadrangle construction still holds if we replace $\sqrt{3}A(B_s \to \pi^0\eta_8)$ with $\sqrt{2}A(B_s \to \pi^0\phi)$, $A(B \to \pi K)$ with $A(B \to \pi K^*)$, and $A(B^+ \to \pi^+\pi^0)$ with $A(B^+ \to \pi^0\rho^+)$.

Deshpande and He[153] predict $B(B_s \to \pi^0\phi) \approx 2 \times 10^{-8}$, which probably means that the quadrangle reduces to two nearly overlapping triangles (whose shapes will consequently be difficult to specify). On the other hand, in PV decays, the effects of electroweak penguins then may not be so important if the dominant processes are characterized by branching ratios of order 10^{-5} as in $B \to PP$ decays.

Hints of some signals have been seen in some PV channels in the latest CLEO data,[145] but only upper limits are being quoted. These are fairly close to theoretical expectations in the case of some $\pi\rho$ channels.

b) VV final states satisfy Bose statistics. Since the total angular momentum of the decaying particle is zero, the (space) $\times$ (spin) part of the VV wave function will be symmetric, as in PP final states.[155] Thus, there should exist amplitude relations for each relative orbital angular angular momentum ℓ. If one ℓ value dominates the decays, such relations might be tested using triangles constructed of square roots of decay rates, as in the PP case.

G.   Progress in studies of CP violation in B physics

Since the earliest observations of B mesons, the goal of utilizing them for CP-violation studies[17,18] has steadily marched toward realization. For perspective, the situation as of January 1987[156] is compared with that of February 1995 in Table 5. Moreover, in recent years there have arisen further possibilities based on symmetric e^+e^- colliders,[130] on the "tagging" of neutral B mesons using hadrons produced nearby in phase space, and on the measurement of CKM phases without explicit detection of CP-violating asymmetries. This area is rich indeed!

6. NON-STANDARD AND SPECULATIVE ASPECTS

A. Superweak theory

It is possible to explain the nonzero value of ϵ in the neutral kaon system by means of an *ad hoc* $\Delta S = 2$ interaction leading directly to CP-violating $K^0 - \bar{K}^0$ mixing.[47] The phase of ϵ will then automatically be the superweak phase mentioned in Section 2, and one will see no difference between η_{+-} and η_{00}. The only evidence against this possibility so far is the $> 3\sigma$ observation of nonzero ϵ'/ϵ by the CERN NA31 experiment,[46] a result not confirmed by Fermilab E731.[45]

A superweak interaction (of considerably greater strength) could in principle lead to observable CP-violating $B^0 - \bar{B}^0$ mixing. If this were so, one would expect[157] $A(\pi^+\pi^-) = -A(J/\psi K_S)$ as a result of the opposite CP eigenvalues of the two final states. In order for this relation to hold in the standard model, one would need $\eta = (1-\rho)[\rho/(2-\rho)]^{1/2}$. Taking account of possible errors in checking that the asymmetries are actually equal and opposite, one concludes that a portion of the allowed region of parameters shown in Fig. 5 could not be distinguished from a superweak theory. The ratio $A(\pi^+\pi^-)/A(J/\psi K_S)$ is informative in a more general context: for example, if it exceeds 1, then ρ must be negative.[97]

If ϵ arises entirely from a superweak interaction, there is no need for CKM phases, and one will see no "direct" effects in kaon or B decays. There will also be no neutron or electron electric dipole moments, though such effects also will be well below experimental capabilities in the standard CKM picture.

B. Right-handed W's

The standard electroweak theory involves coupling of W bosons only to left-handed fermions, through terms in the Lagrangian such as $\bar{d}\gamma^\mu(1 - \gamma_5)W_\mu^{(+)}u$. The left-handed quarks and leptons are members of a doublet of left-handed isospin $\mathrm{SU}(2)_L$, while right-handed quarks and leptons are singlets. The corresponding formula for the charge of a fermion then is $Q = I_{3L} + (Y/2)$, where I_{3L} is the third component of the $\mathrm{SU}(2)_L$ group, while Y is the weak hypercharge [a U(1) quantum number]. The neutral boson W^0 coupling to the SU(2) group and a neutral boson B^0 coupling to the U(1) charge mix with one another to form the photon and the Z^0. There is no need in such a theory for a right-handed neutrino; nothing couples to it. There is a single set of charged W bosons, which we may call $W_L^\pm$. Their leptonic decays are of the form $W \to \ell\nu_\ell$.

The asymmetry of the standard theory has led to the proposal[158] that there also exist *right-handed W's*, coupling to right-handed fermions through terms in the Lagrangian such as $\bar{d}\gamma^\mu(1 + \gamma_5)W_{R\mu}^{(+)}u$. The right-handed quarks and leptons are members of a doublet of right-handed isospin $\mathrm{SU}(2)_R$, while left-handed quarks and leptons are singlets of this group. The fermion charge is now given by the much more symmetric (and easy-to-remember!) formula $Q = I_{3L} + I_{3R} + (B - L)/2$, where B and L are baryon and lepton number: $B = 1/3$ for a quark; $L = 1$ for a lepton.

There are now two neutral bosons W_L^0 and W_R^0 coupling to the two SU(2) groups, and a neutral boson coupling to the $\mathrm{U}(1)_{B-L}$ quantum number. Consequently, there are not one but two Z's in such a theory; one of them must have a mass of least a few hundred GeV in order not to have already been seen. A right-handed neutrino $N_{R\ell}$ (for each lepton family) is needed in the theory for cancellation of triangle anomalies. There are two sets of charged W bosons, $W_L^\pm$ and $W_R^\pm$. The leptonic decay of the W_R must involve the N_R. The N_R and the ordinary left-handed ν_L need not have the same masses if they are *Majorana particles* (equal to their antiparticles), as is permitted for neutral particles. If the N_R turns out to be too heavy to permit $W_R \to \ell N_{R\ell}$, one should look for the W_R instead through the decay $W_R \to t\bar{b}$.[159]

Limits on right-handed W's stem, for example, from the detailed study of the electron spectrum in $\mu \to e\nu\bar{\nu}$.[160] In this manner one concludes that a W_R would have to be heavier than nearly $1/2$ TeV *if right-handed neutrinos are light*. The limits also depend on the degree to which W_L and W_R bosons mix with one another. An even more stringent lower limit of a couple of TeV follows from

the remarkable efficiency of box diagrams (cf. Fig. 3) with one W_L and one W_R in inducing $K - \bar{K}$ mixing.[161] This limit is degraded considerably, however,[162] if the couplings of W_R do not follow the same CKM pattern as those of W_L.

If right-handed W's couple to right-handed fermions, one can obtain new CP-violating interactions (for example, via box diagrams involving W_R and the usual left-handed W_L). The right-handed W mass scale must be tens of TeV or less in order for a large enough contribution to ϵ to be generated. In contrast to the situation described in Section 3, one can generate CP violation using only two quark families, since redefinitions of quark phases are constrained by the right-handed couplings.

An amusing feature of right-handed W couplings is that their participation (or even dominance) in b quark decays is surprisingly hard to rule out.[163] Some suggestions have been made to test the usual picture of left-handed $b \to c$ decays using the polarization of Λ_b baryons produced in b quark fragmentation.[164]

C. Multi-Higgs and supersymmetric models

If there is more than one Higgs doublet, complex vacuum expectation values of Higgs fields can lead to CP-violating effects.[165] It appears that in order to explain $\epsilon \neq 0$ in neutral kaon decays by this mechanism, one expects too large a neutron electric dipole moment.[166] The possibility of such effects remains open, however, and the best test for them remains the study of dipole moments. Current models[167,168] tend to be constrained by the present limits[170] of

$$|d_n| < 1.1 \times 10^{-25} \ e\cdot\text{cm} \ (95\% \text{ c.l.}) \quad , \quad |d_e| < 2 \times 10^{-26} \ e\cdot\text{cm} \ (95\% \text{ c.l.}) \quad . \tag{144}$$

Other CP-violating effects in multi-Higgs-boson models include transverse lepton polarization in $K_{\mu 3}$ decays[169] and various asymmetries in charm decays,[168] which we discuss in Sec. 6 E below.

One needs complex vacuum expectation values in some multi-Higgs models in order to generate CP violation. Supersymmetric models, in which for every particle there is a "superpartner" differing by 1/2 unit of spin, have two Higgs doublets whose ratio of vacuum expectation values is real and thus cannot generate CP violation.[171] However, such models can generate dipole moments at one loop[172] order in perturbation theory. Thus, the possibility of dipole moments showing up just below present bounds is considerably enchanced in such models relative to the standard CKM predictions, which are less than about 10^{-32} $e\cdot$cm for the neutron and 10^{-38} $e\cdot$cm for the electron and rely on contributions at the three-or-greater-loop level.

A useful constraint on multi-Higgs models arises from the $b \to s\gamma$ transition.[173] The observed transition rate[174] corresponding to a branching ratio $B(b \to s\gamma) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}$, is consistent with the standard-model prediction based on the penguin graph,[175] limiting the contributions from other sources such as charged Higgs bosons. Corrections to the standard model's predictions for the $Zb\bar{b}$ vertex (in which loops involving top quarks play a role) also set limits on masses and couplings in multi-Higgs models, while supersymmetric models can improve small discrepancies between experiment and theory for the $Zb\bar{b}$ vertex.[173]

In Fig. 12 we show the constraints on the mass m_+ of the charged Higgs boson and the ratio of vacuum expectation values of the two Higgs doublets, $\tan\beta = v_2/v_1$, from various sources.[173] For large m_+ one has $\tan\beta > 3/4$, while for large $\tan\beta$ the charged Higgs boson is restricted to lie above about 100 GeV.

D. Dipole moments

The Hamiltonian for the interaction of a particle with spin $\mathbf{S}$ with homogeneous electric and magnetic fields can be written

$$h = -(d\,\mathbf{E} + \mu\,\mathbf{B}) \cdot \mathbf{S}/S \quad , \tag{145}$$

where d is the electric dipole moment and μ is the magnetic dipole moment. Under time reversal, $t \to -t$, the fields and spin transform as

$$\mathbf{E} \to \mathbf{E} \quad , \quad \mathbf{B} \to -\mathbf{B} \quad , \quad \mathbf{S} \to -\mathbf{S} \quad , \tag{146}$$
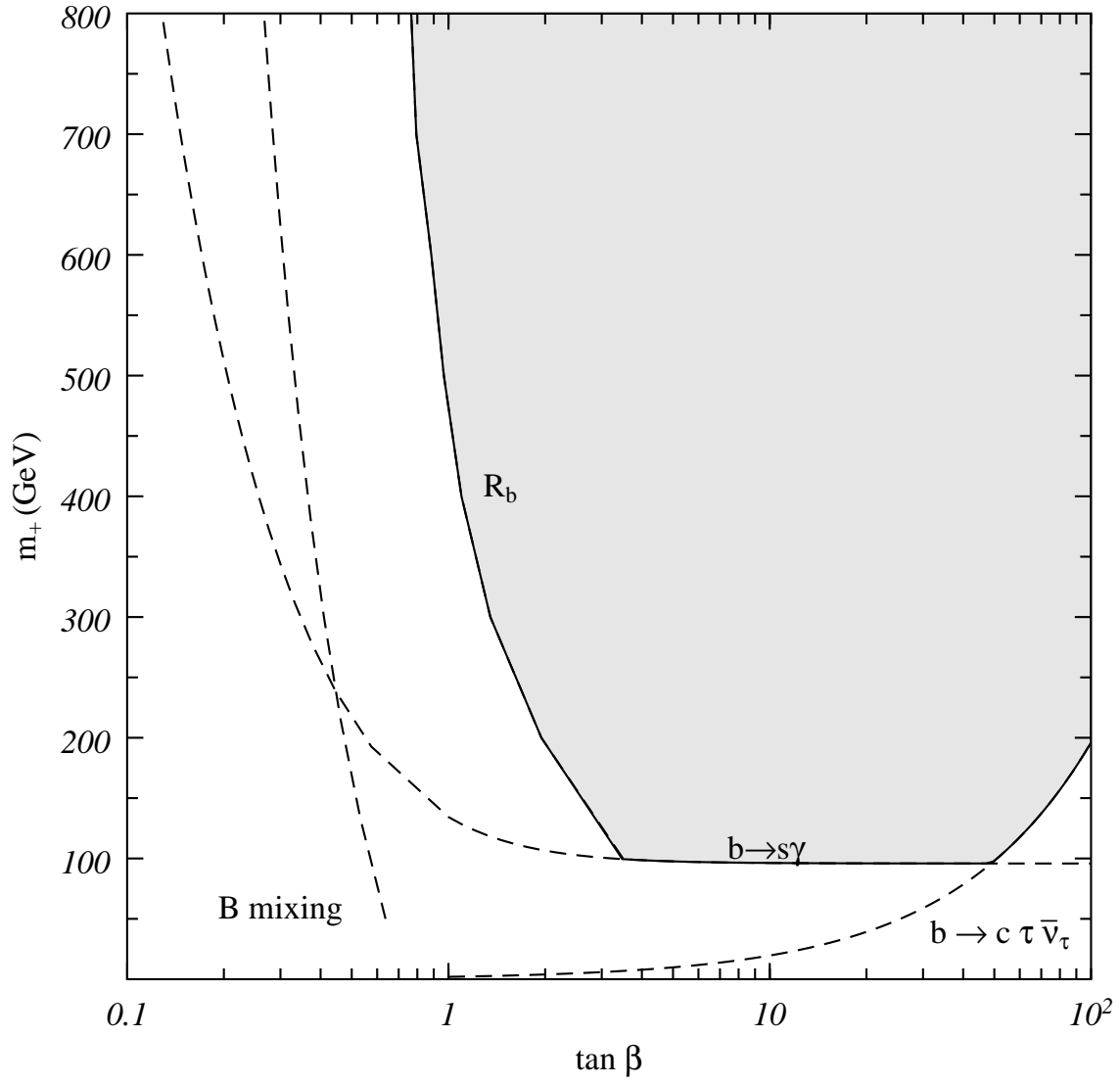
41

Fig. 12. Constraints on two-Higgs models from various experimental sources.[173] The allowed region is shaded. R_b: constraint from the $Zb\bar{b}$ vertex.

Table 6. Rate asymmetries in charmed meson decays.

Charmed meson	Final state	Asymmetry
D^+	$K^- K^+ \pi^+$	-0.031 ± 0.068
D^+	$\bar{K}^{*0} K^+$	-0.12 ± 0.13
D^+	$\phi \pi^+$	0.066 ± 0.086
D^0	$K^+ K^-$	0.024 ± 0.084

so that a nonzero electric dipole moment would violate time reversal invariance. (In fact, a nonzero expectation value $\langle \mathbf{E} \cdot \mathbf{S} \rangle \neq 0$ would violates P as well as T.)

We have mentioned present limits on the neutron and electron dipole moments. The neutron limits are obtained in experiments in which ultra-cold neutrons are confined in a magnetic "bottle" by means of their magnetic moments. The electron limits are based on experiments in atoms, in which fortunate overlaps of levels often lead to substantial amplification of the electron's moment. In the case of the limit cited above, based on atomic thallium, the amplification factor is $d_{\text{Atom}}/d_e \simeq -600$.

An example of a recent stringent limit, based on comparison of Larmor frequencies for parallel and antiparallel electric and magnetic fields,[176] is $d(^{199}\text{Hg}) < 1.3 \times 10^{-27} e \cdot$cm (95% c.l.).

E. Charm decays

The standard model predicts very small CP-violating effects in charmed particle decays. $D^0 - \bar{D}^0$ mixing is expected to be small and uncertain, dominated by long-distance effects.[177] The short-distance contribution to CP-violating mixing should be of order $(m_b/m_t)^2$ times that in neutral kaons, while the lifetime of a neutral D meson is about 0.4 ps in comparison with 52 ns for a K_L. The tree-level decays $c \to su\bar{d}$, $c \to du\bar{d}$, $c \to du\bar{s}$ have zero or negligible weak phases in the standard convention.

For precisely these reasons, CP-violating charmed particle decays are an excellent place to look for new physics.[168,178] New effects tend to be accompanied with flavor-changing neutral currents, which may or may not be an advantage in specific cases. Experiments are easy to perform and undersubscribed in comparison with the many proposed studies of B physics. Information on rate asymmetries $A(f) \equiv [\Gamma(i \to f) - \Gamma(\bar{i} \to \bar{f})]/[\Gamma(i \to f) + \Gamma(\bar{i} \to \bar{f})]$ is just now beginning to appear, as illustrated in Table 6.[179]

F. Baryogenesis, neutrino masses and grand unification

The ratio of baryons to photons in our Universe is a few parts in 10^9. If baryons and antibaryons had been produced in equal numbers, mutual annihilations should have reduced this quantity to a much smaller number,[180] of order a part in 10^{18}. In 1967 Sakharov[22] proposed three ingredients of any theory which sought to explain the preponderance of baryons over antibaryons in our Universe: (1) violation of C and CP; (2) violation of baryon number, and (3) a period in which the Universe was out of thermal equilibrium. Thus our very existence may owe itself to CP violation. However, no consensus exists on a specific implementation of Sakharov's suggestion.

A toy model illustrating Sakharov's idea can be constructed within a model wherein color SU(3) and electroweak SU(2) × U(1) are embedded in a "grand unified" SU(5) gauge group.[181] This group contains "X" bosons which can decay both to uu and to $e^+ \bar{d}$. By CPT, the total decay rates of X and $\bar{X}$ must be equal, but CP-violating rate differences $\Gamma(X \to uu) \neq \Gamma(\bar{X} \to \bar{u}\bar{u})$ and $\Gamma(X \to e^+ \bar{d}) \neq \Gamma(\bar{X} \to e^- d)$ are permitted. This example conserves $B - L$, where B is baryon number (1/3 for quarks) and L is lepton number (1 for electrons).

It was pointed out by 't Hooft[182] that the electroweak theory contains an anomaly as a result of nonperturbative effects. This anomaly conserves $B - L$ but violates $B + L$. If a theory leads to $B - L = 0$ but $B + L \neq 0$ at some primordial temperature T, the anomaly can wipe out any $B + L$ as T sinks below the electroweak scale.[183] Thus, the toy model mentioned above and many others are unsuitable in practice. Proposed solutions include (1) the generation of baryon number directly at the electroweak scale rather than at a higher temperature,[184] and (2) the generation of nonzero $B - L$ at a high temperature, e.g., through the generation of nonzero lepton number L which is then reprocessed into nonzero baryon number by the 't Hooft anomaly mechanism.[185] The first scenario, based on standard model CP-violating interactions (as manifested in the CKM matrix), is widely regarded as inadequate to generate the observed baryon asymmetry at the electroweak scale.[186] We illustrate in Fig. 13 some aspects of the second scenario. (Missing ingredients are denoted by question marks.) The existence of a baryon asymmetry, when combined with information on neutrinos, could provide a window to a new scale of particle physics.

If neutrinos have masses at all, they are much lighter than their charged counterparts or the corresponding leptons. (See Fig. 1.) One possibility for the suppression of neutrino masses[187] is the so-called "seesaw" mechanism, by which light neutrinos acquire Majorana masses of order $m_M = m_D^2/M_M$, where m_D is a typical Dirac mass and M_M is a large Majorana mass acquired by right-handed neutrinos. Such Majorana masses change lepton number by two units and therefore are ideal for generating a lepton asymmetry if Sakharov's other two conditions are met.

The question of baryogenesis is thus shifted onto the leptons: Do neutrinos indeed have masses? If so, what is their "CKM matrix"? Do the properties of heavy Majorana right-handed neutrinos allow any new and interesting natural mechanisms for violating CP at the same scale where lepton number is violated? Majorana masses for right-handed neutrinos naturally violate left-right symmetry and could be closely connected with the violation of P and C in the weak interactions.[188]

An open question in this scenario, besides the precise form of CP violation at the lepton-number-violating scale, is how this CP violation gets communicated to the lower mass scale at which we see CKM phases. Presumably this occurs through higher-dimension operators which imitate the effect of Higgs boson couplings to quarks and leptons.

The presence of a suitable mass scale for lepton number violation is suggested by certain patterns of electroweak-strong unification. If the strong and electroweak coupling constants are evolved to high mass scales in accord with the predictions of the renormalization group,[189] as shown in Fig. 14(a), they approach one another in the simplest SU(5) model,[181] but do not really cross at the same point. This "astigmatism" can be cured by invoking supersymmetry,[190] as illustrated in Fig. 14(b). Here the cure is effected not just by the contributions of superpartners, but by the richer Higgs structure in supersymmetric theories. The theory predicts many superpartners below the TeV mass scale, some of which ought to be observable in the next few years.

Alternatively, one can embed SU(5) in an SO(10) model,[191] in which each family of quarks and leptons (together with a right-handed neutrino for each family) fits into a 16-dimensional spinor representation. Fig. 14(c) illustrates one scenario for breaking of SO(10) at two different scales, the lower of which is a comfortable scale for the breaking of left-right symmetry and the generation of right-handed neutrino Majorana masses.

G.   The strong CP problem

We will be very brief about this subject. There are fine reviews elsewhere.[19,192] As a result of nonperturbative effects, the QCD Lagrangian acquires an added CP-violating term $g_s^2 \bar{\theta} F_{\mu\nu}^a \tilde{F}^{\mu\nu a}/32\pi^2$, where $\bar{\theta} = \theta + \text{Arg det } M$, θ is a term describing properties of the QCD vacuum, and M is the quark mass matrix. The limit on the observed neutron electric dipole moment, together with the estimate[19] $d_n \simeq 10^{-16}\bar{\theta}$ e cm, implies that $\theta \leq 10^{-9}$, which looks very much like zero. How can one undertand this? Several proposals exist.

1. Vanishing m_u. If one quark mass vanishes (the most likely candidate being m_u), one can rotate away any effects of $\bar{\theta}$.[19] However, it is generally though not universally felt that this bends the
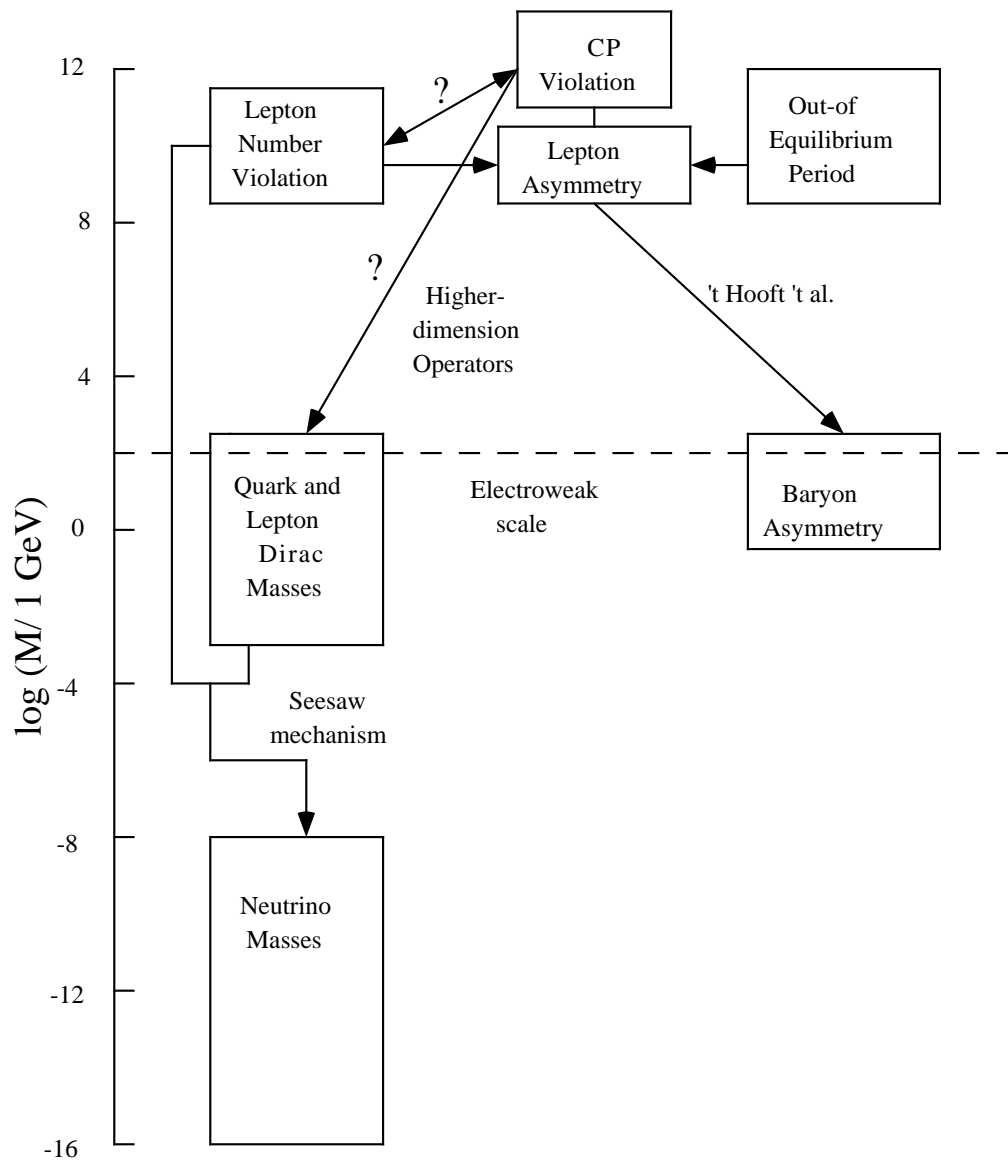
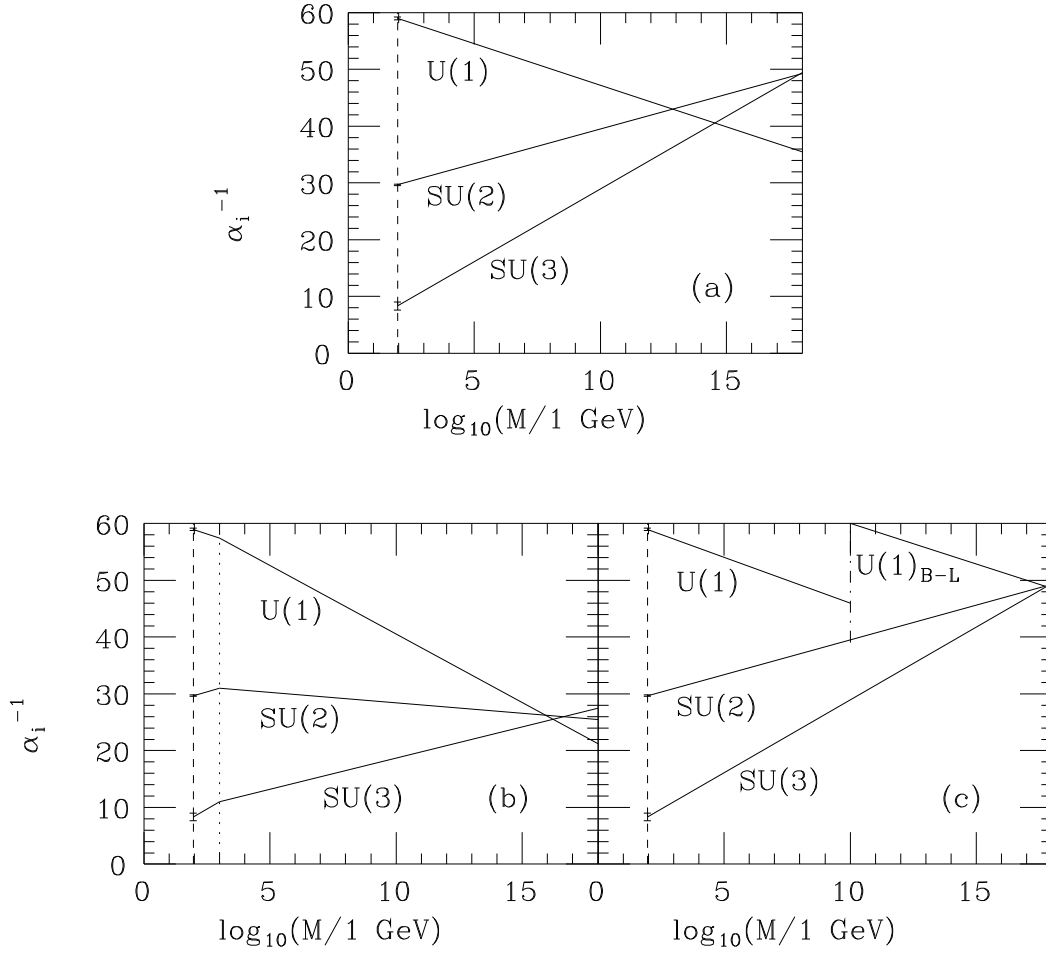Fig. 13. Mass scales associated with one scenario for baryogenesis.

Fig. 14. Behavior of coupling constants predicted by the renormalization group in various grand unified theories. Error bars in plotted points denote uncertainties in coupling constants measured at $M = M_Z$ (dashed vertical line). (a) SU(5); (b) supersymmetric SU(5) with superpartners above 1 TeV (dotted line) (c) example of an SO(10) model with an intermediate mass scale (dot-dashed vertical line).

constraints of chiral symmetry beyond plausible limits.[193] My own guess is that light-quark masses are in the ratios $u : d : s = 3 : 5 : 100$.

2. Axion. One can introduce a continuous U(1) global symmetry[20] such that $\bar{\theta}$ becomes a dynamical variable which relaxes to zero as a result of new interactions. The spontaneous breaking of this symmetry then leads to a pseudo-Nambu-Goldstone boson, the *axion*,[21] for which searches may be performed in many ways. My favorite is via the Primakoff effect,[194] in which axions in the halo of our galaxy interact with a static man-made strong magnetic field to produce photons with frequency equal to the axion mass (up to Doppler shifts). These photons can be detected in resonant cavities. Present searches would have to be improved by about a factor of 100 to detect axions playing a significant role in the mass of the galaxy.[192]

3. Boundary conditions. It has been proposed[195] that one consider not the θ-vacuum, but an incoherent mixture consisting of half θ and half $-\theta$, in the manner of a sum over initial spins of an unpolarized particle. The experimental consequences of this proposal are still being worked out.

H.   Possible origin of the CKM Matrix

Numerous attempts have been made to understand the structure of the CKM matrix by imposing some discrete ("family") symmetries on the quarks and leptons. One thereby obtains strategically placed zeroes in the mass matrices, which can lead to relations between quark masses and CKM matrix elements. We have reviewed such approaches previously.[24] Here we would like to mention a favorite class of models which is considerably less popular, but worthy of consideration nonetheless.

We imagine quarks as being composite objects. States of definite mass and charge (such as the quarks d, s, and b) are imagined to involve mixing of configurations of the constituents. Thus we envision families as corresponding to orthogonal mixed configurations. There should be a gap between the light-fermion families and additional states at the compositeness scale.[196]

Let us imagine that a weak charge-changing transition changes the identity of one of the constituents, thereby altering its interaction with its neighbors. As a consequence, a rotation will be induced in the basis of states, so that u, c, and t will involve different mixed configurations than d, s, and b. The simplest example in the quark model is provided by the lowest-lying states of the charmed baryon $\Xi_c^+ = csu$ or $\Xi_c^0 = csd$, which correspond approximately (but not exactly) to basis states in which the two light quarks are coupled to spin 0 or 1. If $m_u \neq m_d$, the Ξ_c^+ and Ξ_c^0 will be slightly different mixtures of the basis states.[197] This two-family system thus provides an illustration of the origin of the Cabibbo angle.

We have constructed a toy model of the CKM matrix[198] with the above mechanism in mind. One gets satisfactory relations between CKM elements and quark masses, but at the price of discrete choices of interactions between the subunits. There is no obvious source of the CKM phase in such a model; it has to be put in by hand. The interested reader is referred to the original article for more details.

I.   Overview of nonstandard and speculative aspects

Each of the suggested alternatives to the standard picture of CP violation may be tested against particular experimental touchstones. A key experiment to disprove the superweak model of CP violation in the kaon system will be the observation of a nonzero value of ϵ'. The scale at which right-handed W's make their appearance, if at all, is inversely correlated with neutrino masses in "seesaw" types of models for those masses. Multi-Higgs models for CP violation are constrained by existing limits on electric dipole moments, so that such effects could show up at any time as experimental sensitivities are improved.

The standard model of CP violation predicts very small effects in charmed particle decays. Since charmed particles are easier to produce than B's and the standard expectations for CP asymmetries in their decays are small, they are a very good place to look for non-standard effects.

The scenario we have presented for baryogenesis, based on its correlation with lepton number violation, again hinges on the values of neutrino masses. We have proposed that fundamental CP and lepton number violation arise together at a high mass scale, for which neutrino masses would provide an estimate. The strong CP problem may be addressed experimentally by means of axion searches, of which those based on RF cavities are particularly appealing. Even the possibility that quarks and leptons are composite can be addressed experimentally, though the problems in such models are well-known and serious.[196,199]

7. CONCLUSIONS

The observed CP violation in the neutral kaon system has been successfully parametrized in terms of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The problem has been shifted to one of understanding the magnitudes and phases of CKM elements. Even before this more ambitious question is addressed, however, one seeks independent tests of the CKM picture of CP violation. Rare K decays and B decays will provide many such tests.

Alternative (non-CKM) theories of CP violation are much more encouraging for some CP-violating quantities like the neutron electric dipole moment or effects in charmed particle decays. However, most of these alternative theories do not predict observable direct CP-violating effects in K or B decays.

No real understanding exists yet of baryogenesis or of the strong CP problem. Fortunately, there exist many possibilities for experiments bearing on these questions, including searches for neutrino masses and for axions.

The CKM picture suggests that we may understand CP violation better when the pattern of fermion masses itself is understood. Why is the top quark heavier than all the other quarks and leptons (or why are the others so much lighter than the top?) The observation of the top quark[78,79] has forced us to confront this problem, and perhaps we will figure out the answer some day.

Acknowledgments

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